ToppersNotes

IES/GATE

CIVIL ENGINEERING

FLUID MECHANICS

VOLUME-II

Sierra Innovations Pvt. Ltd.

Contents

Fluid Mechanics

Mechanics

⇓

Study of motion

(Kinematics) study of motion without the consideration of basic causes of motion i.e. force.

$\vec{v} = \frac{d\vec{s}}{dt}$

$\vec{a} = \frac{d\vec{v}}{dt}$

$\vec{j} = \frac{d\vec{a}}{dt}$

Jerk.

not including mass (directly or indirectly)

unit – m, m/s, m/s², m/s³

Dynamics – study of motion with the cosideration of basic causes of motion. i.e. force.

$\vec{F}_{ex} = \frac{d}{dt}(m\vec{v})$

including mass.

Dynamic Viscosity (μ) = $\frac{N\text{-}s}{m^2}$

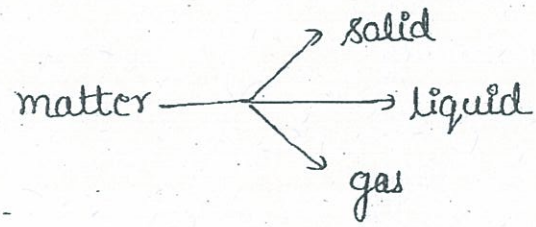
kinematic viscosity (ν) = $\frac{\mu}{\rho}$ (m^2/s)

Fluid Mechanics :-

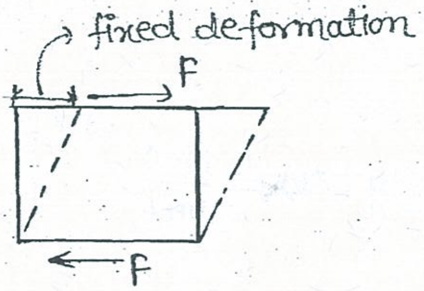
Fluid :- "Liquid & Gases both are having the property of contineous deformation under the action of shear or tongential force. This property of contineously deformation is also known as flow property & Hence Liquid & gases are kept in diffrent catagory which is far away from the solids & this catagory is known as fluid."

A fluid is a substance which is having & ability to flow under the action of shear & tongential forces.

Fluid —



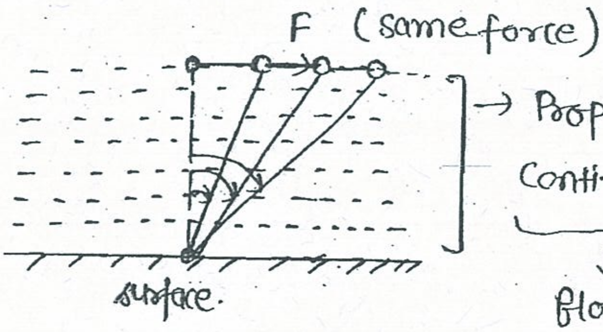
In solid →



deformation change when forces are changes at different-2 time.

In liquid :-

At same force, deformation are changes contineously.



→ Property of Contineous deformation

⇓

flow property.

Fluid as a Continuum :—

"In macroscopic system, the inter atomic space b/w the molecules of fluid can be treated as neglegible as compared to the dimension of the system therefore we can assume adjacent to one molecule there is a another molecule. & there is no interspace b/w them. Hence the entire fluid molecule system can be treated as contineous distribution of mass system & it is known as continuum."


flow.

BASIC FLUID PROPERTY :—

(i) **Density (ρ)** :— It is defined as mass per unit body of the substance.

$$\boxed{\rho = \frac{m}{V}}$$

unit :- kg/m^3.

In C.G.S unit —

$$1 gm/c.c = 1 gm/cm^3.$$

$$= \frac{10^{-3} kg}{10^{-6} m^3} = \frac{1000 kg}{m^3}.$$

(2) **Specific Weight** :- It is the weight of the substance per unit volume.

$$sp.\ wt. = \frac{mg}{V} = \rho . g.$$

$$\boxed{sp.\ wt. = \rho g} \quad N/m^3. \qquad \frac{F}{L^3}$$

(3) Specific Gravity (s.g) :- A sp. gravity of a fluid is defined as a Ratio of density of fluid to the density of standard fluid.

$$\boxed{(s.g)_{fluid} = \frac{\text{Density of fluid}}{\text{Density of standared fluid}}}$$

for lq. ⇒ Standard fluid ⇒ water ($1000\ kg/m^3$).

for gas ⇒ Standard fluid ⇒ Atm. Air ($1.21\ kg/m^3$).

(4) Relative density (R.D) :-

$$\boxed{(R.D.)_{1/2} = \frac{\rho_1}{\rho_2}}$$

(5) Compressibility (β) :-

$$\boxed{\beta = \frac{-\frac{d\forall}{\forall}}{dP}} \quad \text{—— (1)}$$

$$\boxed{m = \rho \times \forall = \text{Constant}}$$

$$\rho \cdot d\forall + \forall \cdot d\rho = 0$$

$$\boxed{\frac{-d\forall}{\forall} = \frac{d\rho}{\rho}}$$

Put these value in eq^n (1)

$$\boxed{\beta = \frac{1}{\rho}\cdot\frac{d\rho}{dP}}$$

If ρ is not changing w.r.t pressure –

$$\frac{d\rho}{dP} \to 0 \Rightarrow \boxed{\beta = 0}$$

Incompressive.

If ρ is changing w.r.t pressure –

$$\frac{d\rho}{dP} \not\to 0 \Rightarrow \boxed{\beta \neq 0}$$

Compressive.

Liquid

$\Downarrow$

Compressible

For water :–

at 1 atm $\longrightarrow$ $\rho_{water} = 998\ kg/m^3$

at 100 atm $\longrightarrow$ $\rho_{water} = 1003\ kg/m^3$.

$\therefore\ \Delta\rho = 5\ kg/m^3$

$$\%\ change = \frac{5}{998}\times 100 = \frac{\Delta\rho}{\rho}\times 100$$

$$\approx 0.5\%$$

$$\boxed{\beta_{liq} = 0}$$

Liquid are treated as Incompressible.

Gases

$\Downarrow$

Highly Compressible

$$P = \rho RT$$

$$\boxed{P \propto \rho}$$

NOTE :- The Reciprocal of compressibility is known as Bulk modulus of elasticity.

(6) Isothermal Compressibility of gas :-

$$\boxed{\beta = \frac{1}{\rho} \cdot \frac{d\rho}{dP}}$$

Ideal gas eq^n —

$$P = \rho RT$$

$$\rho = \frac{P}{RT}$$

[Isothermal T = Constant].

$$\boxed{\frac{d\rho}{dP} = \frac{1}{RT}}$$

$$\therefore \boxed{\beta_{iso} = \frac{1}{\rho} \cdot \frac{1}{RT}}$$

$$\boxed{\beta_{iso} = \frac{1}{\rho RT}}$$

$$\boxed{\beta_{iso} = \frac{1}{P}}$$

$$\boxed{K_{iso} = \frac{1}{\beta_{iso}} = P}$$

(7) Adiabatic Compressibility of gas :-

$$\boxed{\beta = \frac{1}{\rho} \cdot \frac{d\rho}{dP}}$$

Adiabatic eqn —

$$P V^{\gamma} = \text{Constant}$$

$$\left[\rho = \frac{m}{V} \qquad V = \frac{m}{\rho} \right]$$

$$P \cdot \frac{m^{\gamma}}{\rho^{\gamma}} = \text{Constant}$$

$$\therefore \quad P \rho^{-\gamma} = \text{Constant}$$

$$P(-\gamma)\, \cancel{\rho^{-\gamma}}\rho^{-1} d\rho + dP \cdot \cancel{\rho^{-\gamma}} = 0$$

$$dP = \frac{\gamma P}{\rho} \cdot d\rho$$

$$\frac{d\rho}{\rho} = \frac{dP}{\gamma P}$$

$$\boxed{\frac{d\rho}{dP} = \frac{\rho}{\gamma P}}$$

$\boxed{\gamma - \text{gama}}$

$$\therefore \quad \boxed{\beta_{Adia} = \frac{1}{\rho} \times \frac{\rho}{\gamma \cdot P} = \frac{1}{\gamma \cdot P}}$$

$$\boxed{\beta_{Adia} = \frac{1}{\gamma . P}}$$

$$\boxed{K_{adia.} = \gamma . P}$$

$$\boxed{\gamma_{Air} = 1.4}$$

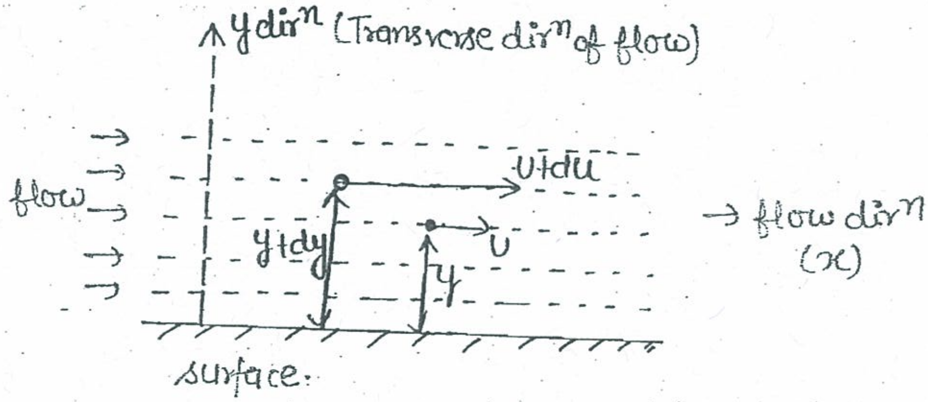
VISCOSITY

"The two adjacent layer of fluid resist the motion of each other such a fundamental property of fluid is known as viscosity."

Basic Reason of Viscosity :- Cohesion ⇒ for liquid.
intermolecule attraction

In gases ⇒ Cohesion (negligible)

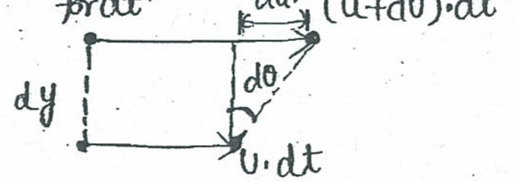
$(\text{viscosity})_{gases} <<<< (\text{viscosity})_{liquid}$



The Relative velocity of the contacting layer = zero.
(No-slip CONDITION)

⇒ There will be the development of velocity gradient in transverse dir^n of flow $\left(\frac{du}{dy}\right)$.

In the fig →



Shear deformation b/w adjacent layer =

$$\tan d\theta = \frac{du \cdot dt}{dy}.$$

$$\boxed{d\theta = \frac{du \cdot dt}{dy}.}$$

Rate of shear deformation →.

$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}.}$$

"Rate of shear deformation is same as velocity gradient"
$d\theta/dt$ · du/dy

Newton's Law of Viscosity

The shear stress b/w the adjacent layer at distance y from the surface will be--

$$\tau \propto \left(\frac{d\theta}{dt}\right)$$

$$\boxed{\tau = \mu\left(\frac{d\theta}{dt}\right)}$$

Constant (But not universal)
⇓
Property of fluid.
as well as depend upon temp.

If μ → high →

$$\Rightarrow \frac{d\theta}{dt} \rightarrow \text{less}$$

Flow is difficult.

$\Rightarrow$ If $\mu \rightarrow$ less

$$\frac{d\theta}{dt} = \text{high}$$

flow is easy.

$\mu \Rightarrow$ $\boxed{\text{Direct measurement of internal Resistance b/w the layers of fluid.}}$

$\Downarrow$

dynamic viscosity.

Dynamic Viscosity (μ) :-

Unit :-

$$\mu = \frac{\tau}{\left(\frac{d\theta}{dt}\right)} = \frac{\tau}{\left(\frac{du}{dy}\right)} = \frac{N}{m^2 \cdot \frac{m}{s \cdot m}} = \frac{N\text{-}s}{m^2}$$

S.I UNIT :-

$\hookrightarrow \frac{N\text{-}s}{m^2} \Rightarrow (Pa\text{-}s)$

M.K.S Unit :-

$\hookrightarrow \frac{kg\text{-}m\text{-}s}{s^2 \cdot m^2} = \frac{kg}{m\text{-}s}$

$$\boxed{\frac{1kg}{m\text{-}s} = 1\,Pa\text{-}s}$$

C.G.S Unit →

Poise

$$1\ \text{Poise} = \frac{1gm}{cm\text{-}s}$$

$$= \frac{10^{-3}\ kg}{10^{-2}\ m\text{-}s} = 0.1\ Pa\text{-}s$$

Kinematic viscosity :- (ν)

$$\boxed{\nu = \frac{\mu}{\rho}}$$

UNIT → m^2/s.

C.G.S UNIT — Stoke.

$$\boxed{\begin{aligned} 1\,\text{stokes} &= \frac{1\ cm^2}{s} \\ &= 10^{-4}\ m^2/s \end{aligned}}$$

Effect of temp. on the Viscosity of the fluid :-

" Basic Reason of Viscosity — Cohesion.

$(\text{cohesion})_{gas} \Rightarrow$ Almost nill.

$$\boxed{\mu_{gas} <<<< \mu_{lq}}$$

But $\nu_{gas} = \dfrac{\mu_{gas}}{\rho_{gas}}$

It may be $\nu_{gas} > \nu_{lq}$

$\nu_{gas} < \nu_{lq}$

$\nu_{gas} = \nu_{lq}$.

Liquid :-

If $T \uparrow \Rightarrow (\text{Cohesion})_{lq} \downarrow$.

$\Rightarrow (\mu)_{lq} \downarrow$.

$$\nu_{lq} = \frac{\mu_{lq}}{\rho_{lq}} \Rightarrow \downarrow.$$

If $T \uparrow \Rightarrow (\mu)_{lq} \downarrow$ as well as $(\nu)_{lq} \downarrow$.

But Rate of $\downarrow$ in $(\mu)_{lq}$ is None.

Gas :-

Cohesion is almost nil.

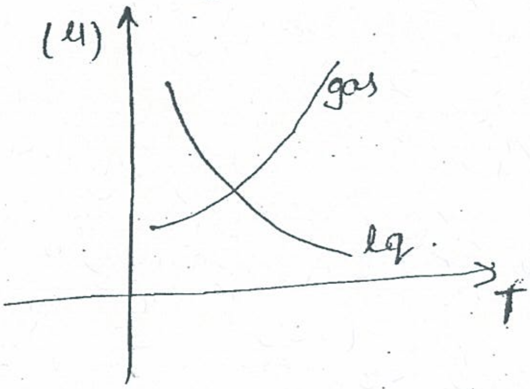
$$\bar{C}_{r.m.s} = \sqrt{\frac{3RT}{M}} \propto \sqrt{T}$$

If $T \uparrow \Rightarrow \bar{C}_{rms} \uparrow$

$\Rightarrow$ Randomness $\uparrow$

$\Rightarrow$ It will introduce some additional Resistance in path of fluid flow.

$\Rightarrow \mu_{gas} \uparrow$.



for kinamatic –

$$\nu_{gas} = \frac{\mu_{gas}}{\rho_{gas}}.$$

$P = \rho RT$

$\rho \propto \frac{1}{T}$.

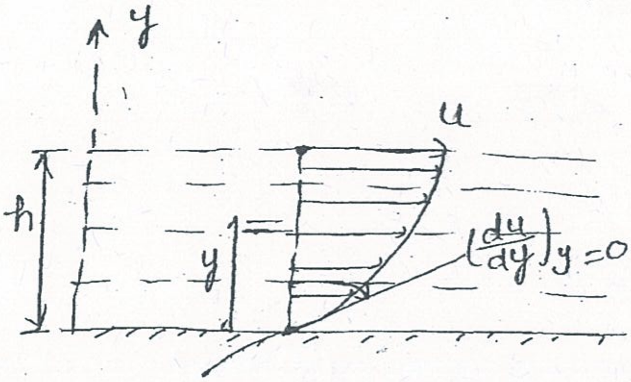
with

$T \uparrow \Rightarrow \nu_{gas} \uparrow$

If $T \uparrow \Rightarrow \mu_{gas} \uparrow$ & $\nu_{gas} \uparrow$.

But Rate of $\uparrow$ is more in ν_{gas}.

Lineraxization of Newton's Law of Viscosity :-



Acc. to N. Law of viscosity –

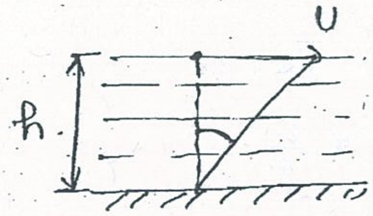
$$\tau = \mu\left(\frac{du}{dy}\right)$$

shear stress at the surface

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}.$$

when h is very-2 small of the order of mm.

⇒ Velocity profile can be treated as st. line.



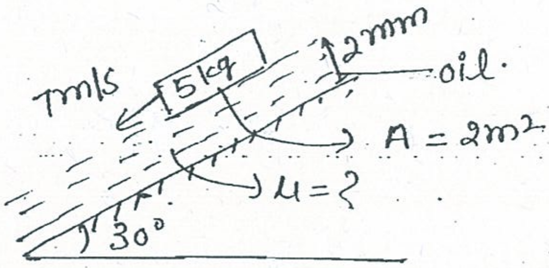
$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$$= \mu\left(\frac{U-0}{y}\right) = \frac{\mu U}{y}.$$

3) drag force. $\Rightarrow$ $\boxed{F_{drag} = \frac{\mu U}{h} \cdot A}$ $\rightarrow$ surface Area.

Numerical

Pb. 1
2 marks.



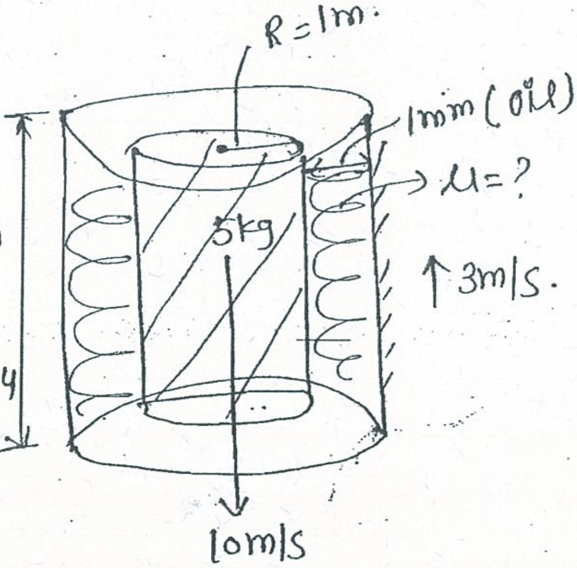
Net force —
$F = 0$

$5g \sin 30^\circ = F_{Drag}$

$5 \times 9.81 \times \frac{1}{2} = \frac{(7-0)}{\cancel{2} \times 10^{-3}} \times (\mu) \times \cancel{2}$ $\Rightarrow 24.525 = 7 \times 10^3 \mu$

$\boxed{\mu = 3.5 \times 10^{-3}} \frac{Ns}{m^2}$

Pb-2
2-marks



$5g = F_{Drag}$.

$5 \times 9.81 = \mu \left[\frac{10-(-3)}{1 \times 10^{-3}}\right] \times 2 \times \pi \times 1 \times 4$

$\boxed{\mu = 1.5 \times 10^{-4}}$

Pb-3
10 marks

(i) find
(i) y such that drag on moving plate from both of the fluid is same.