ToppersNotes

IES/GATE
CIVIL ENGINEERING

STRUCTURE ANALYSIS
RAILWAY HIGHWAY

VOLUME-VIII

Sierra Innovations Pvt. Ltd.

Contents

Structural Analysis

- IES
  - Obj — 15-20 question
  - Conv. — 40 to 60 marks
- GATE — 8 to 10 marks

Syllabus

→ Stability and degree of Indeterminacy →(for obj.)

→ Influence line diagram → (obj + conv.)

→ Arches → (obj)

→ Method of Analysis
  - → Strain Energy method
  - → Slope deflection method
  - → Moment distribution method
  - → Matrix Method

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→ Analysis of Trusses
  - → Determinate Trusses
  - → Indeterminate Trusses
  - → Deflection of Truss joints

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Reference Books

(1). Theory of Structures by S. Ramamurtham
(for strain energy method, moment distribution and truss analysis)

(2). Structure Analysis by Negi & Jangid.
(for slope deflection & Influence line diagram and strain energy method).

(3). Theory of structures by Gupta & Pandit
(Basic concept of stability and determinacy, also good for matrix analysis)

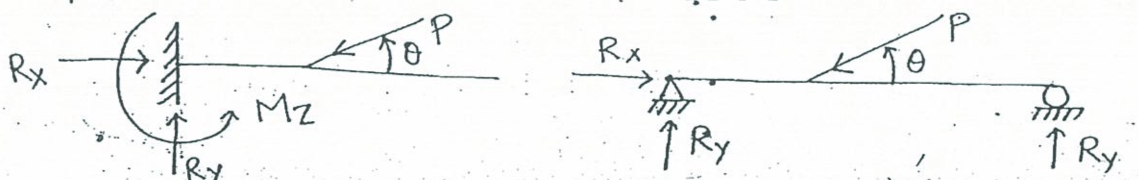
(4). Structure Analysis by R.C. Hibler
(for truss analysis & basics of structures)

Stability and Indeterminacy

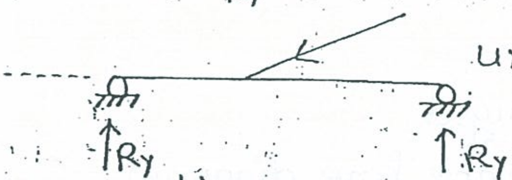
Stability
- External: related to support conditions.
- Int.: Related to geometry.

External stability. Large displacements of the supports or entire structure are not ~~permitted~~, therefore there should be enough reactions at supports to prevent movement & also reactions should be arranged in appropriate manner. It means there will not be rigid body-motion, however elastic deflection in the member may occur.

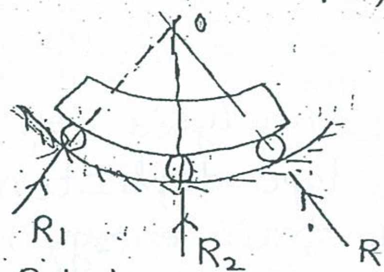
In plane structures (2D) there should be a min.m of 3 independent external reactions which should be non-parallel and non-concurrent



unstable



Linear unstability



Rotational unstability

For stability of 2D structure forming three condition of static equilibrium should be satisfied.

(i) $\sum F_x = 0$ - - - - To prevent Δ_x

(ii) $\sum F_y = 0$ - - - To prevent Δ_y

(iii) $\sum M_z = 0$ - - - - To prevent θ_z.

In case of 3D structure, there should be a min.m of independent ext. reactions to prevent rigid body displacements at support.

The displacements to be prevented are,

Δ_x , Δ_y , Δ_z , θ_x , θ_y , θ_z

Therefore, there will be 6 equations of static equilibrium.

$\Sigma F_x = 0$ $\quad \Sigma F_z = 0$ $\quad \Sigma M_y = 0$

$\Sigma F_y = 0$ $\quad \Sigma M_x = 0$ $\quad \Sigma M_z = 0$

2D structures are called plane structures & 3D structures are called space structures. In 3D struct for stability all the reactions should be non-coplanar/ non-parallel and non-concurrent.

Internal Stability - No part of the structure can move relative to the other part, so that geometry of the structure is preserved. However, small elastic deformations are permitted. To preserve the geometry, enough no. of members and adequate arrangement is required. For geometric stability, there should not be formation of condition of mechanism. It means there should not be 3 co-linear hinges.

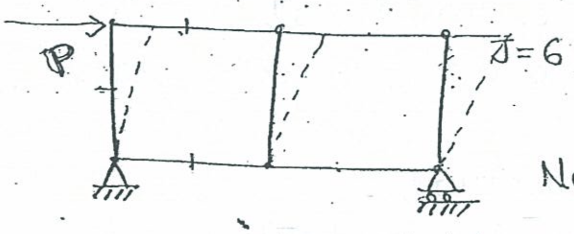
For 2D truss, the min^m no. of members neede for geometric stability is,

$$\boxed{m = 2j - 3}$$

and for 3D truss,

$$\boxed{m = 3j - 6}$$

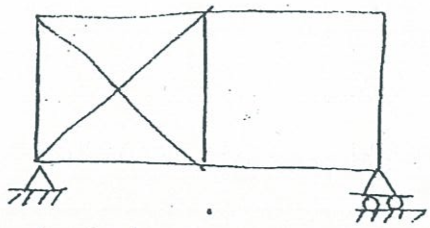
All the members should be arranged such tha truss is divided in triangular blocks, there should not be rectangular or polygonal blocks.



No. of members required for stability = $2j - 3 = 6 + 3 = 9$

No. of members provided = 7.

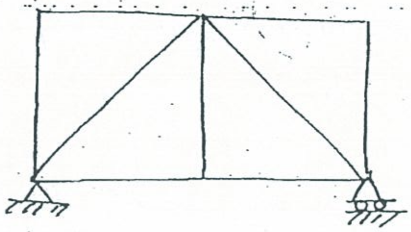
Hence, structure is having internally unstability condition and it is called



$m = 9$

$= 2j - 3$

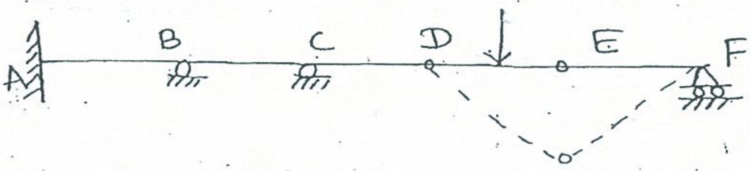
In above case arrangement of members is not adequate, Hence, right panel is unstable and left panel is over-stiff. For geometric stability all ~~plaplanet~~ panels of the truss should be stable.



$m = 9$

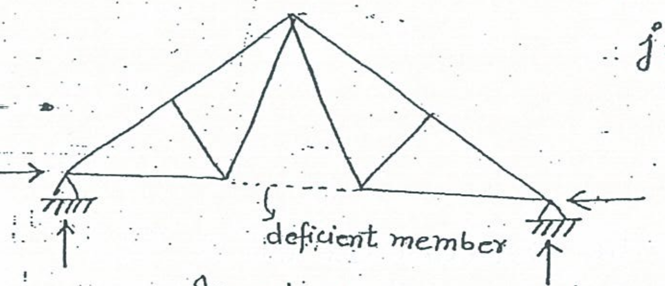
$= 2j - 3 = 2 \times 6 - 3 = 9$

Stable / Perfect



Mechansism.
3 collinear Hinges.

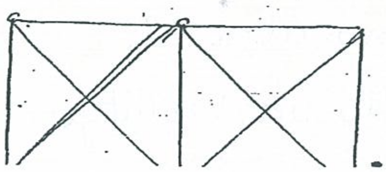
Overall Stability - For overall stability, ext. stability is compulsory. In some cases structure is overall stable but it may be overstiff externally & deficient internally, it means support rxns. are more than 3 and no. of truss members are less than $2j - 3$.



$j = 7$, $2j - 3 = 11$

$\therefore m = 10$

In above case, truss is overall stable because there is one extra redundant reaction which prevent geometric deficiency. It is desirable for overall stability, structure should be externally and internally stable both.



In this structure,
externally unstable &
Internally stable but

Indeterminacy (D_i)

- Static ($D_{si} + D_{se} = D_s$) related to support conditions
  - External (D_{se})
  - Internal (D_{si})
- Kinematic (D_k) related to geometry & D.O.F.

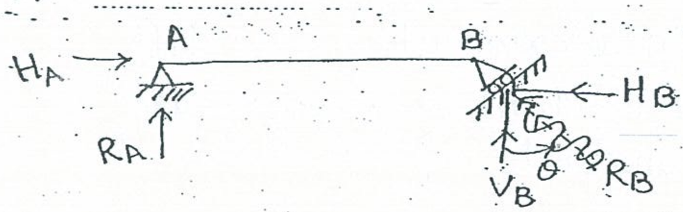
Static Indeterminacy- Those structures which cann't be analysed using conditions of static equilibriu alone, are called indeterminate structures (OR) hypersta structures. For indeterminate structures, to analyse additional ~~equilibrium~~ conditions are required, called compatibility conditions.

(1)- External static indeterminacy (D_{se}) - It is related t support system of the structure and it is equ to no. of independent ext. rxns in ~~access~~ excess to available equilibrium conditions for stable equilibrium.

Let, r_e = Total no. of support reactions (Independer

$D_{se} = r_e - 3$ For 2D

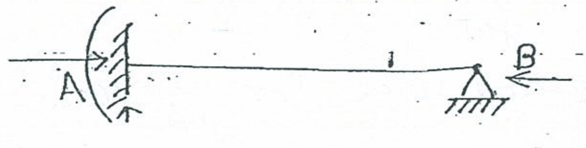
$= r_e - 6$ For 3D



$R_B \cos\theta = V_B$
$R_B \sin\theta = H_B$ } dependen

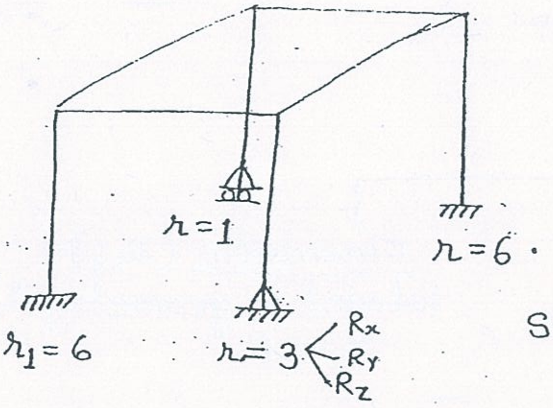
r_e < H_A, R_A, R_B [Independent]

Question- For the structure shown in figure, determine $D_{se} = ?$



$r_e = 5$

$D_{se} = 5 - 3 = 2$



Total $r_e = 6+6+4$
$= 16$

$D_{se} = r_e - 6$
$= \underline{10}$

Stable and Indeterminate.

Internal Static Indeterminacy (D_{si}) -

Case I - Pin jointed plane frame (2 D truss)

In trusses all joints are hinged and loading is applied only at joints & self wt. is ignored. Hence, all truss members will carry only axial forces. If there are 'm' members in the truss, hence there will be m internal reactions (axial forces). At each joint in the truss, there are two equilibrium condition ($\Sigma F_x = 0$ & $\Sigma F_y = 0$). Let, there are 'j' no. of joints, hence total equilibrium conditions at all joints $= 2j$

Out of '2j' equations, three equations are used at supported joint to determine external rxns. Hence, net available eqns. to determine internal rxns $= (2j-3)$

Therefore,

$$\boxed{D_{si} = m - (2j-3)}$$

If, $D_{si} = 0$, Internally determinate (Perfect Trusses)

$D_{si} > 0$, Internally indeterminate & overstiff

$D_{si} < 0$, Internally Deficient & geometrically unstable.

se II - Three D Truss (Pin jointed space frame)

In 3D truss also each member has 1 internal reaction i.e. axial force but each joint has 3 eqns. of equilibrium ($\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$). Therefore, total equilibrium equations at 'j' joint $= 3j$

∴ Out of these, 6 equations are used for external

$$\boxed{D_{si} = m - (3j - 6)}$$

Case III- 2D Rigid frames and 3D Rigid frames

In rigid frames, internal indeterminacy will not exi if it forms an open configuration like a tree. To check internal indeterminacy, following thumb rules can be applied -

If structure is internally determinate, then it is impossible to make a cut anywhere on the structure without splitting the structure in two parts.

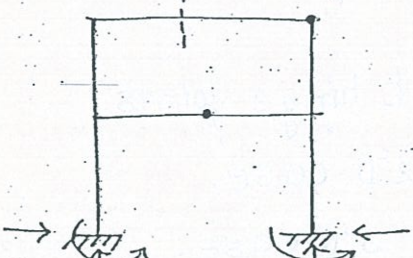
In case of internally determinate structure, it is impossible to return back at same point without retracing the path, it means internally determinate structures do not have cyclic loops.

In 2D rigid members, each member has 3 internal reactions (R_x, R_y and M_z) (OR) (S.F., Axial Thrust & B.M.) whereas in 3D rigid members, each membe has 6 internal reactions ($R_x, R_y, R_z, M_x, M_y, M_z$). It me each closed loop in 2D has 3 internal indetermina and in 3D, it has 6 internal indeterminacy.

$\boxed{D_{si} = 3C}$ for 2D Rigid frame

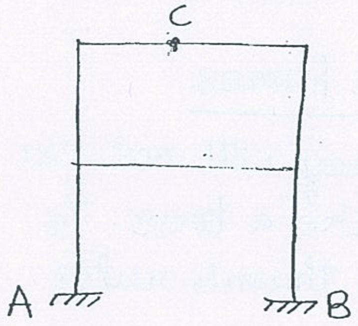
$= 6C$ for 3D ,, ,,

C = No. of closed loops.



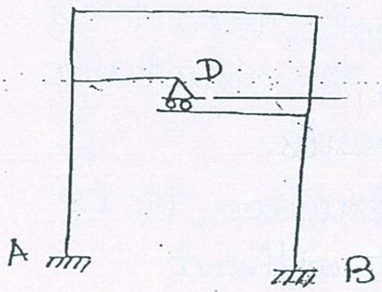
In above analysis, all the joints are considered rigi If some of the joints are hybrid (hinge), then some of the internal reactions will be released. Hence, Dsi will be reduced. Hence for hybrid frames,

$D_{si} = 6C - r_r$ for 3D case.



Hence, one internal reaction (M_c) is released.

$r_r = 1$.



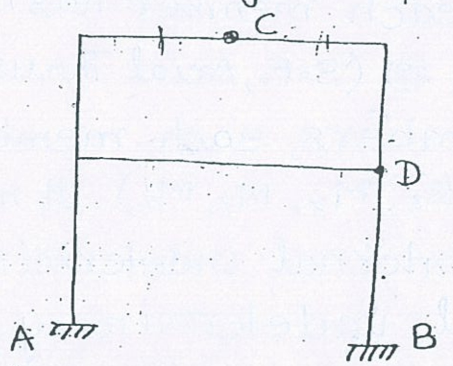
① Axial Force

② Bending Moment

$r_r = 2$

The no. of released reactions depend upon no. of members meeting at hybrid joints.

e.g.→



At C, one internal reaction is released.

At D, two internal reactions are released.

$r_r = 1 + 2 = 3$

For plane structures,

$$r_r = \text{summation } (m'-1) = \sum (m'-1)$$

For space structures,

$$r_r = \sum 3(m'-1)$$

where, m' = No. of members meeting at hinge joints.

$$D_{si} = 3C - \sum (m'-1) \quad \text{for 2D case.}$$

$$= 6C - \sum 3(m'-1) \quad \text{for 3D case.}$$

Overall degree of static indeterminacy (D_s),

$$\boxed{D_s = D_{se} + D_{si}}$$

= External Indeterminacy + Int. Indeterminacy

Alternative approach to find overall (D_S) -

Case I → 2 D Truss (plane truss)

$$D_S = m + R_e - 2j$$

where,
m = no. of int. rxns
R_e = no. of ext. support rxns.
$2j$ = total available equilibrium equations

If $D_S = 0$, Truss is statically determinate
$D_S > 0$, Truss is statically indeterminate
$D_S < 0$, Truss is statically unstable.

Case II → 3D Truss (space truss)

$$D_S = m + R_e - 3j$$

Case III → 2D Rigid Frame

$$D_S = 3m + R_e - 3j$$

(when all joints are rigid.)

$$D_S = 3m + R_e - 3j - r_r$$

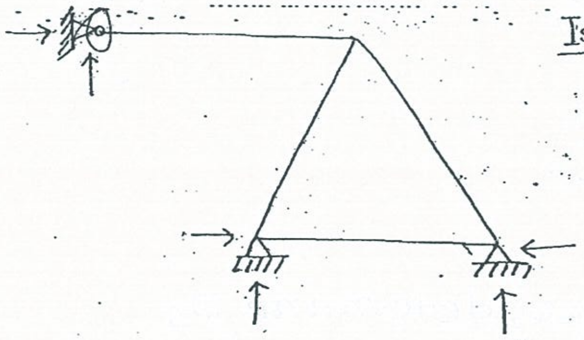
(when joints are hybrid.)

Case IV → 3D Rigid Frame

$D_S = 6m + R_e - 6j$ (when all joints are rigid).

$D_S = 6m + R_e - 6j - r_r$ (when joints are hybrid).

Example - For 2D truss shown in figure, find $D_S = ?$



Ist Method,

$D_{Se} = 6 - 3 = 3$

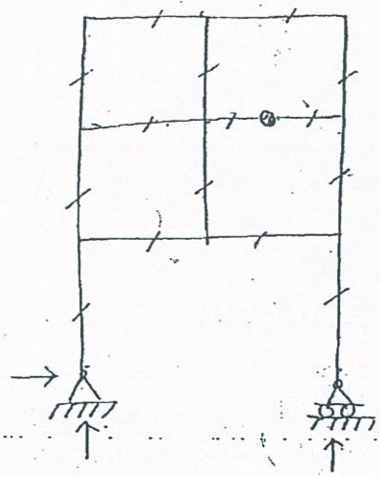
$D_{Si} = m - (2j - 3)$
$= 4 - 2 \times 4 + 3$
$= -1$

$\therefore D_S = 3 - 1 = 2$

IInd Method,

$D_S = m + R_e - 2j$
$= 4 + 6 - 8 = 2$

3)→ For 2D frame shown in figure, find D_S = ?



$D_{Se} = 3-3 = 0$

$D_{Si} = 3C - r_r$

$= 3\times4 - 1$

$= 11$

$D_S = D_{Si} + D_{Se} = \underline{11}$

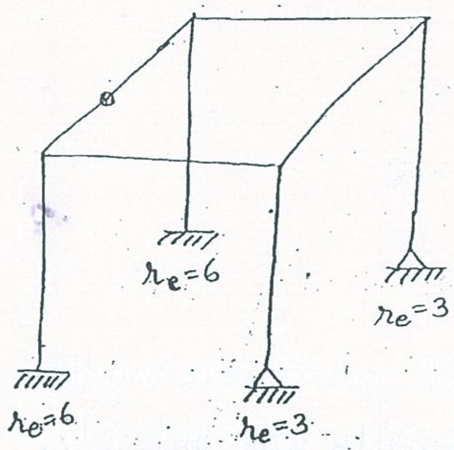
second method

$D_S = 3m + r_e - 3j - r_r$

$= 3\times15 + 3 - 3\times12 - 1$

$= 48 - 37 = \underline{11}$

8)→ For 3D hybrid frame shown in figure, find D_S,



Total, $R_e = 6 + 6 + 3\times2$

$= 18$

$r_r = 3(m' - 1)$

$= 3(2-1) = 3.$

$D_{Se} = 18 - 6 = 12$

$D_{Si} = 6C - r_r$

$= 6\times1 - 3 = 3$

$D_S = D_{Si} + D_{Se} = \underline{15}$

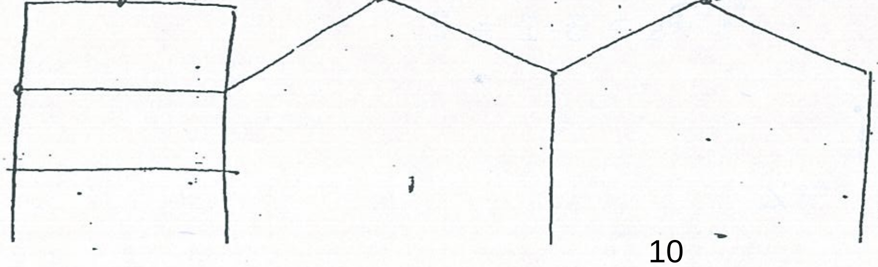
second method

$D_S = 6m + R_e - 6j - r_r$

$= 6\times9 - 6\times9 + 18 - 3$

$= \underline{\underline{15}}$

)→ For rigid frame shown in figure, determine D_S,



$r_e = 12$

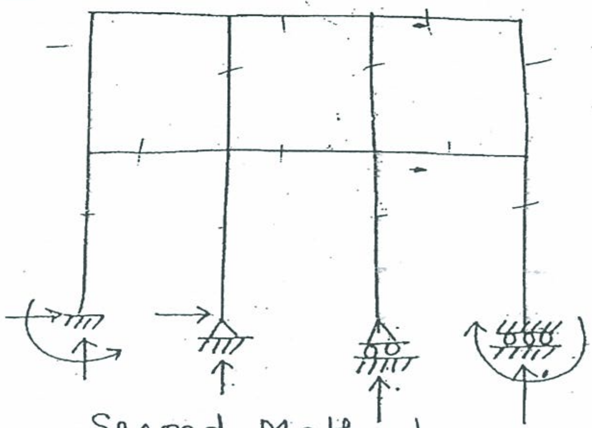
$D_{Se} = 12 - 3 = 9$

$$D_{si} = 3C - r_r$$
$$= 3\times 2 - 5 = 1$$
$$D_s = D_{si} + D_{se} = 9+1 = \underline{\underline{10}}$$

Second method

$$D_s = 3m + r_e - 3j - r_r$$
$$= 3\times 16 + 12 - 3\times 15 - 5$$
$$= \underline{10}$$

(Q)→ Find the overall $D_s = ?$

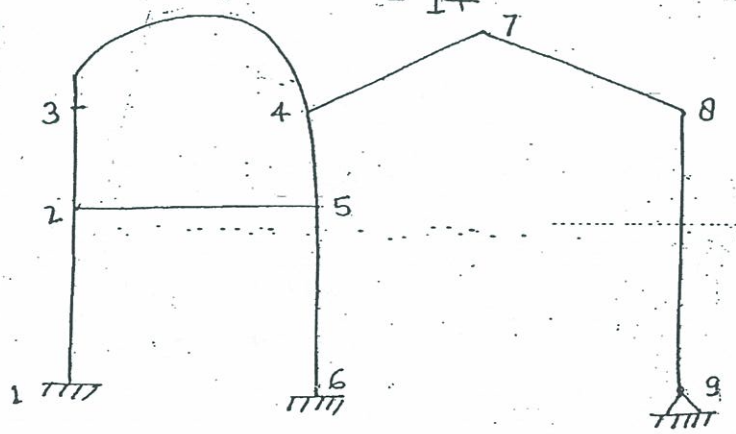


$$D_{se} = 8 - 3 = 5$$
$$D_{si} = 3C - r_r$$
$$= 3\times 3 - 0$$
$$= 9$$
$$D_s = D_{si} + D_{se} = \underline{\underline{14}}$$

Second Method

$$D_s = 3m + r_e - 3j - r_r$$
$$= 3\times 14 + 8 - 3\times 12 - 0$$
$$= 14$$

(Q)→



$$D_{se} = r_e - 3$$
$$= 3\times 2 + 2 - 3$$
$$= 5$$
$$D_{si} = 3C - r_r$$
$$= 3\times 1 = 3$$
$$\therefore D_s = 5 + 3 = \underline{8}$$

Second method

$$D_s = 3m + r_e - 3j - r_r$$
$$= 3\times 9 + 8 - 3\times 9$$
$$= \underline{\underline{8}}$$

Degree of Kinematic Indeterminacy.

It refers to the total no. of available degree of freedom at all joints.

(OR)

It is equal to total no. of unrestrained displacement components at all joints.

D_K = Total no. of D.O.F. available at all joints

Type of Joint	Possible no. of D.O.F.
1). 2D Truss joint	2 — Δx, Δy
2). 3D Truss joint	3 — Δx, Δy, Δz
3). 2D Rigid joint	3 — Δx, Δy, θz
4). 3D Rigid joint	6 — Δx, Δy, Δy, θx, θy, θz

Type of Structure	D_K
2D Truss (Plane truss)	$= 2j - R_e$
3D Truss (space truss)	$= 3j - R_e$
2D Rigid frame	$= 3j - R_e$
3D Rigid frame	$= 6j - R_e$

In above analysis, all members are considered axially flexible & all above displacements are elastic displacement.

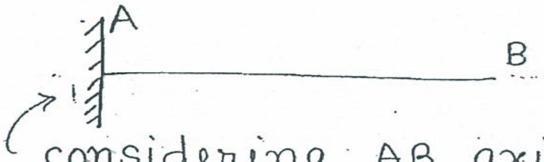
Case I → In rigid frames, if some of the members are axially rigid, then in such members, axial displacement may not be available. Hence, D_K will be reduced.

For 2D rigid frames,

$$\boxed{D_K = 3j - R_e - m''}$$

m' is no. of axially rigid members.

In above case, if axial displacement is already not present, then no need to substract.



$D_K = 3\times2-3 = 3 < \Delta_{bx}, \Delta_{by}, \theta_{bz}$

considering AB axially flexible.

If AB is axially rigid, then D_K will be,

$$D_K = 3\times2-3-1 = 2 < \Delta_{by}, \theta_{bz}$$

consider axially flexible,

$$D_K = 3\times2-4 = 2 < \theta_A, \theta_B$$

consider axially rigid,

$$D_K = 3\times2-4 = 2 < \theta_A, \theta_B$$

∵ because there is no axial displacement.

Case II → If in rigid frames, some of the joints are hybrid then additional degrees of freedom will be available. Hence, increase in D_K will be equal to no. of reactions released at hybrid joint.

$$\boxed{D_K = 3j - r_e - m'' + r_r}$$

For 2D Rigid frame with hybrid joints.

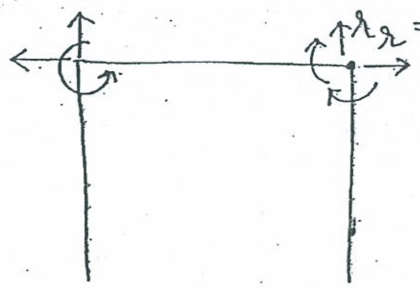
j = Total no. of joint (Rigid + Hybrid) & supported + unsupported.

$r_r = \Sigma(m'-1)$

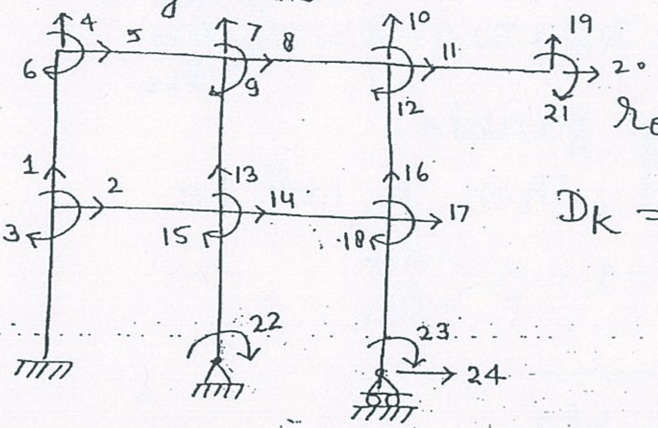
For, 3D Rigid frame,

$$D_K = 6j - r_e - m'' + r_r$$

$r_r = \Sigma 3(m'-1)$



(Q)> For 2D Rigid frame shown in figure, find D_K and show all displacement components at respective joints.



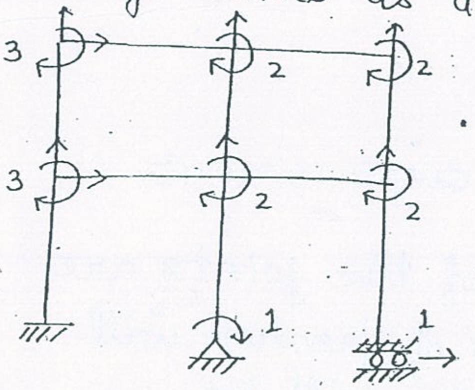
$r_e = 3+1+2 = 6$

$D_K = 3j - r_e$

$= 3\times10 - 6$

$= \underline{\underline{24}}$

Q)> For rigid jointed frame shown in figure, find D_K. assuming beams as axially inextensible.



$m'' = 4$

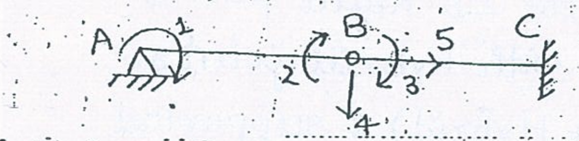
$r_e = 3+2+2 = 7$

$D_K = 3j - R_e - m''$

$= 3\times9 - 7 - 4$

$= 20 - 4 = \underline{\underline{16}}$

)> For beam shown in figure, find D_K considering eam as flexible.



$D_K = 3j - R_e - \cancel{m''} + r_r$

$= 3\times3 - 5 - 0 + 1$

$= \underline{\underline{5}}$

If above beams are axially rigid, then near axial displacement components at joint B ll not be available. ⇒

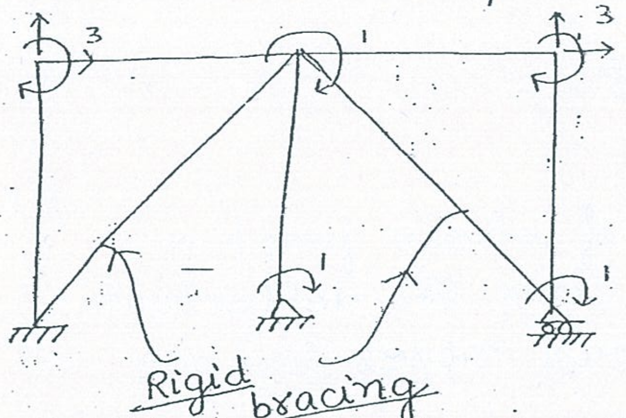
$D_K = 4$

$\theta_A \quad \theta_{B_2}$

$\theta_{B_1} \quad \Delta_{By}$

$\boxed{\Delta B_x = 0}$

→ In braced frame shown in figure, find degree of kinematic indeterminacy.

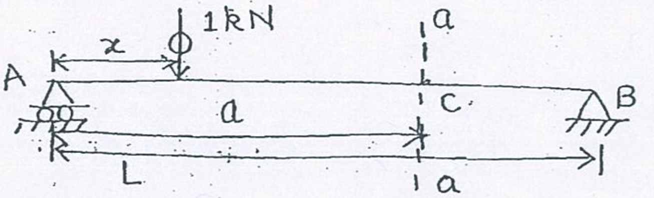


$D_K' = 3+1+3+1+1$

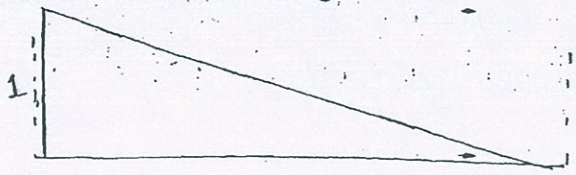
$= \underline{\underline{9}}$

Braced members are rigid which prevent linear displacement component at braced joints.

Influence Line Diagram

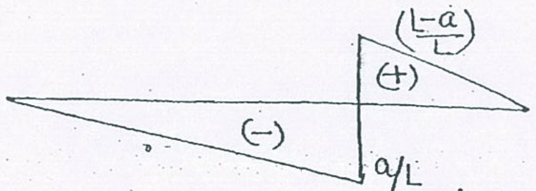


ILD represents variation of stress function
h as S.F., B.M., reaction, slope (OR) deflection
a point when unit conc. load moves from one
d to the other end.

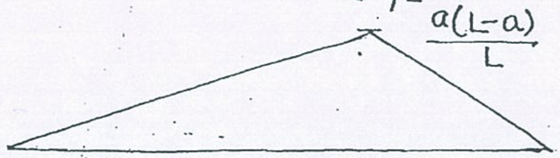


ILD for R_A

ILD for R_B



ILD for SF_c



ILD for $B.M_c$

ses/applications of I.L.D.,

.D can be used to study the effect of a moving
l on the structure.

.D can be used to find position of live load
ch will produce maximum value of a particular
ess function.

.D can be used to calculate total value of a
ticular stress function due to a given load system