Toppersnotes

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CSIR-NET

Council of Scientific & Industrial Research

PHYSICAL SCIENCE

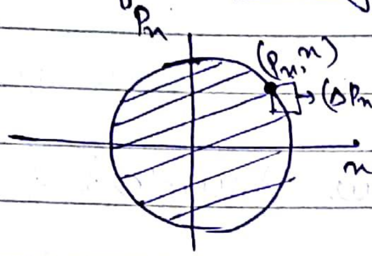
VOLUME - V

QUANTUM MECHANICS

SIERRA INNOVATIONS

QUANTUM MECHANICS

Bohr Somerfield Theory :-



i) $\oint p_x dx = nh$

area under the phase space is discreat.

Hygenberg says we cann't take it as a point because it has some minimum value Δx and Δp_x so in Q.M.

ii) $\boxed{\Delta x \cdot \Delta p_x \geq \hbar/2}$

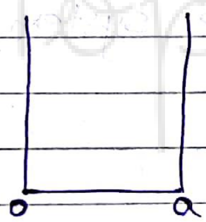
So smallest area will be $\Delta x \cdot \Delta p_x$

(iii) De broglie says for every moving particle there is a wave associated particle. so. $\lambda = h/mv$

$\Rightarrow$ Q.M. is only applicable for the dynemicals.

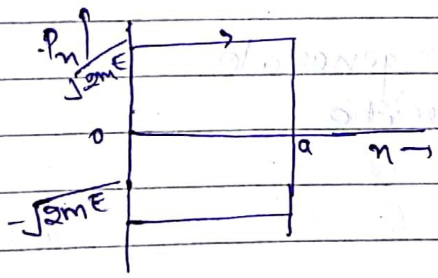
Means where is momentum for a particle we use. Q.M.

(1)



$E = \frac{p^2}{2m}$ $\quad p_x \to E \to 0$

$\oint p_x dx = nh$



$A = 2\sqrt{2mE} \times a = h$

$\boxed{E = \frac{h^2}{8ma^2}}$ - min value of energy. $(n=1)$

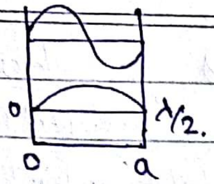
$\boxed{E = \frac{n^2h^2}{8ma^2}}$ $\quad \boxed{E \propto n^2}$

(ii) $E = \frac{p_x^2}{2m} = \frac{(\Delta p_x)^2}{2m}$ (acc. to hygenberg)

$\Delta x \cdot \Delta p_x = \hbar/2$ $\quad (\Delta x)_{max} = a$

$E_{min} = \frac{\hbar^2}{2m \cdot 4(\Delta x)^2}$ $\quad (\Delta p_x)_{min} = \frac{\hbar}{2(\Delta x)}$

$= \frac{\hbar^2}{8ma^2}$

(iii) 

$$\lambda = \frac{h}{p_x} \qquad E = \frac{p_x^2}{2m}$$

$$E = \frac{h^2}{2m\lambda^2} \qquad \lambda/2 = a$$

$$\boxed{E = \frac{h^2}{8ma^2}}$$

Q. Use the Bohr Somerfield theory prove that energy of harmonic oscillator is proportional to n.

Tools of Q.M. :-

$$\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$
$$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$
$$\vec{A} + \vec{B} = \vec{C} = (a_1+b_1)\hat{i} + (a_2+b_2)\hat{j} + (a_3+b_3)\hat{k}$$
$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$
$$\vec{A} + (-\vec{A}) = \vec{0}$$
$$(a+b)\vec{A} = a\vec{A} + b\vec{A}$$
$$a(\vec{A}+\vec{B}) = a\vec{A} + a\vec{B}$$
$$ab\vec{A} = ba\vec{A}$$

$a_1 = \hat{i}\cdot\vec{A}$	$\hat{i}\cdot\hat{i} = 1$	$\hat{i}\cdot\hat{j} = 0$	Orthogonal-perpendicular to
$a_2 = \hat{j}\cdot\vec{A}$	$\hat{j}\cdot\hat{j} = 1$	$\hat{j}\cdot\hat{k} = 0$	each other
$a_3 = \hat{k}\cdot\vec{A}$	$\hat{k}\cdot\hat{k} = 1$	$\hat{k}\cdot\hat{i} = 0$	normal - unit length.

Only linear independent terms make basis.

Hilbert Space :-

$H(x)$: $\phi(x), \psi(x), \xi(x), \chi(x), a, b, c, d, e$

All are $\phi(x), \psi(x), \xi(x), \chi(x)$ are fu^n of Hilbert space. They can be real or complex.

For same Hilbert space all are fu^n of same thing.

If ϕ is member of Hilbert Space & ψ is also member of same hilbert space the $\phi+\psi = \xi$ is also a member of same hilbert space.

$$(\phi+\psi) = \psi+\phi$$
$$(\phi+\psi)+\xi = \phi+(\psi+\xi)$$
$$\phi+(-\phi) = 0$$

character of Scalar- $(a+b)\psi = a\psi + b\psi$

$$a(\psi+\phi) = a\psi + a\phi$$

$ab\psi = ba\psi$

$0\cdot\psi = 0$

The members of Hilbert space will follow vector addition scalar addition and scalar multiplication rule.

$\vec{A}$

Product Rule :-

$\vec{A}\cdot\vec{B} = |A||B|\cos\alpha$

Scalar product b/w ψ & ϕ $\rightarrow (\psi, \phi)$

$$= \int_{-\infty}^{\infty} \psi^*(x)\,\phi(x)\,dx = \text{finite no.}$$

$$(a_1\phi_1 + a_2\phi_2 , b_1\psi_1 + b_2\psi_2) = \int_{-\infty}^{\infty} (a_1^*\phi_1^* + a_2^*\phi_2^*)(b_1\psi_1 + b_2\psi_2)\,dx$$

$$= a_1^* b_1 \int_{-\infty}^{\infty} \phi_1^*\psi_1\,dx + a_1^* b_2 \int_{-\infty}^{\infty} \phi_1^*\psi_2\,dx + a_2^* b_1 \int_{-\infty}^{\infty} \phi_2^*\psi_1\,dx + a_2^* b_2 \int_{-\infty}^{\infty} \phi_2^*\psi_2\,dx$$

$$= a_1^* b_1 (\phi_1, \psi_1) + a_1^* b_2 (\phi_1, \psi_2) + a_2^* b_1 (\phi_2, \psi_1) + a_2^* b_2 (\phi_2, \psi_2)$$

Square Integrable :-

$$(\psi, \psi) = \int_{-\infty}^{\infty} \psi^*\psi\,dx = \int_{-\infty}^{\infty} |\psi|^2 dx = \alpha$$

$\alpha \rightarrow$ +ve finite no.

then ψ is said to be square integrable fun. For graph $|\psi|^2$ in Area under the curve will be finite no.

The square integrable fun must converge at $x \to +\infty$ as well as $x \to -\infty$.

Ex - $\psi(x) = Ae^{ikx}$ $\quad -\infty < x < \infty$

$|\psi|^2 = \psi^*\psi = |A|^2$

$$\int_{-\infty}^{\infty} |\psi|^2 dx = |A|^2 \int_{-\infty}^{\infty} dx$$ — its not a finite no. so $\psi(x)$ is not square integrable.

$$\psi(x) = Ae^{ikx} \qquad 0 < x < a$$
$$0 \qquad \text{otherwise}$$

$$\int_{-\infty}^{\infty} |\psi|^2 dx = \int_{-\infty}^{0} 0 + \int_0^a |A|^2 dx + \int_a^{\infty} 0$$

$$= \underline{\underline{|A|^2 a}}.$$

Now if $\psi(x)$ is bounded it treated as square integrable

Ex- $\psi(x) = \cos x$

$$\int_{-\infty}^{\infty} \cos^2 x \, dx = \cancel{2}\int_0^{\infty} (1 + \cos 2x)\, dx = 1 + \frac{\sin 2x}{2}\Big|_0^{\infty} = \infty$$

Not finite

Square integrability can be defined by fn as well as the value of variables.

$$(\psi(x), \psi(x)) = \int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

then ψ is said to be normalized. Any square integrable fn can be normalized by

$$\frac{1}{\alpha}\int |\psi|^2 dx = \frac{\alpha}{\alpha} = 1.$$

$$\psi_N = \frac{\psi}{\sqrt{\alpha}}$$

$$\boxed{\psi_N = \frac{\psi}{\sqrt{\alpha}} = \frac{\psi}{\sqrt{\int_{\text{all space}} |\psi|^2 dx}}}$$

$\sqrt{\int \psi^* \psi \, dx}$ = Norms of the fn (this will always +ve)

Orthogonality cond^n :- If we have two diff. fu^n -

$$\boxed{(\phi,\psi) = \int \phi^* \psi \, dx = 0}$$

then we can say ϕ and ψ are orthogonal.

Orthonormal cond^n :-

$$(\phi_i,\phi_j) = \int \phi_i^* \phi_j^* \, dx = \delta_{ij} \begin{cases} 1 & i=j \rightarrow \text{Normal cond}^n \\ 0 & i \neq j \rightarrow \text{orthogonal cond}^n \end{cases}$$

Linear Independence :-

$$\phi_1, \phi_2, \phi_3 \,---\, \phi_N$$
$$C_1, C_2, C_3 \,---\, C_N$$

$$C_1\phi_1 + C_2\phi_2 + C_3\phi_3 --- C_N\phi_N = 0$$

If $C_1 = C_2 = C_3 --- C_N$ all values are zero. then this values fu^{n}s are independent to each other.

Ex- the $\phi_1 = x$ $\quad \phi_2 = 5x$

$$C_1 x + C_2 5x = 0$$
$$C_1 = -5C_2 \rightarrow C_1 = 0, \; C_2 = 0$$
$$C_1 = 1, \; C_2 = 5$$

they are dependent to each other.

Ex- $\phi_1 = x$, $\phi_2 = x^2$

$$C_1 x + C_2 x^2 = 0$$
$$x(C_1 + C_2 x) = 0.x + 0.x^2$$
$$\cancel{C_1} = -\cancel{C_2 x} \quad C_1 = 0, \; C_2 = 0$$

this is unique value not any other value will satisfy this eq^n.

Ex - $\phi_1 = e^{-\alpha x^2}$ $\phi = x e^{-\alpha x^2}$

$$c_1 e^{-\alpha x^2} + c_2 x e^{-\alpha x^2} = 0$$

$$e^{-\alpha x^2}(c_1 + c_2 x) = 0$$

$$e^{-\alpha x^2} \quad c_1 + c_2 x = 0 + 0.x \Rightarrow c_1 = 0, \; c_2 = 0$$

Unique solⁿ. Independent to each other.

If $\phi_1, \phi_2, \phi_3 --- \phi_N$ is linearly independent fuⁿ of the same Hilbert space then any fuⁿ ψ can be written as

$$\boxed{\psi = \sum_n a_n \phi_n}$$

$$\left.\begin{array}{l} c_1\phi_1 + c_2\phi_2 + --- + c_N\phi_N = 0 \\ c_1 = c_2 = c_3 --- c_N = 0 \end{array}\right]$$ Condⁿ at basis.

$$(\phi_m, \phi_n) = \delta_{mn}$$

$$\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{A} = (\hat{i}.\vec{A})\hat{i} + (\hat{j}.\vec{A})\hat{j} + (\hat{k}.\vec{A})\hat{k}$$

$$\psi = a_1\phi_1 + a_2\phi_2 + a_3\phi_3 --- \phi_N\phi_N$$

$$a_1 = (\phi_1, \psi)$$

$$a_2 = (\phi_2, \psi) \quad --- \quad a_N = (\phi_N, \psi)$$

$$\boxed{\psi = \sum_n (\phi_n, \psi)\phi_n}$$

unit vector (normalized)

Dirac Notation :-

bra ket

$\langle \; | \; \rangle$

$\psi \rightarrow |\psi\rangle$ ket $\rightarrow \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} \rightarrow |\psi\rangle$

$\psi^* \longrightarrow \langle\psi|$ bra

$$\langle\psi| = (c_1^* \; c_2^* \; c_3^* \; c_4^*)$$

$$|\psi\rangle = \begin{pmatrix} 1+i \\ 2i \\ 3 \end{pmatrix} \qquad \langle\psi| = (1-i \quad -2i \quad 3)$$

$$(\phi, \psi) = \int \phi^*(x)\,\psi(x)\,dx \Rightarrow \langle\phi|\psi\rangle$$

$$(a_1 \; a_2 \; a_3) \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = a_1c_1 + a_2c_2 + a_3c_3$$

$$\langle Q_i | Q_j \rangle = \delta_{ij} \; \begin{matrix} = 1 & i = j \\ = 0 & i \neq j \end{matrix}$$

Ex

$$|\psi_1\rangle = a_1|u_1\rangle + a_2|u_2\rangle$$

$$|\psi_2\rangle = b_1|u_1\rangle + b_2|u_2\rangle$$

$$\langle u_i | u_j \rangle = \delta_{ij}$$

What will be condition such that ψ_1, ψ_2 are orthonormal

Sol^n

$$\langle\psi_1|\psi_2\rangle = a_1^*\langle u_1| + a_2^*\langle u_2| \; \Big| \; b_1|u_1\rangle + b_2|u_2\rangle$$

$$= a_1^*b_1\langle u_1|u_1\rangle + a_1^*b_2\langle u_1|u_2\rangle + a_2^*b_1\langle u_2|u_1\rangle + a_2^*b_2\langle u_2|u_2\rangle$$

$$= a_1^*b_1 + a_2^*b_2$$

ψ_1, ψ_2 are orthogonal then $a_1^*b_1 + a_2^*b_2 = 0$

and ψ_1 and ψ_2 are normalized then $|a_1|^2 + |a_2|^2 = 1$

$\langle\psi_1|\psi_1\rangle = 1$ $\qquad |b_1|^2 + |b_2|^2 = 1.$

$\langle\psi_2|\psi_2\rangle = 1$

$$|\psi_1|^2 = \psi_1^* \psi_1 = (a_1^* u_1^* + a_2^* u_2^*)(a_1 u_1 + a_2 u_2)$$

$$= |a_1|^2 |u_1|^2 + |a_2|^2 |u_2|^2 + a_1^* a_2 u_1^* u_2 + a_2^* a_1 u_2^* u_1$$

$$= |a_1|^2 |u_1|^2 + |a_2|^2 |u_2|^2 + 2\,\mathrm{Re}\,(a_1^* a_2 u_1^* u_2)$$

$$\left(f(x) + f^*(x) = 2\,\mathrm{Re}\, f(x) \right)$$

$|\psi\rangle\langle\phi| \longrightarrow$ Matrices or operator.

<u>Operator</u> :-

$$A|\phi\rangle = |\psi\rangle$$

$$\langle\phi| A^\dagger = \langle\psi|$$

$$(CA)^\dagger = C^* A^\dagger$$

$$(A^\dagger)^\dagger = A$$

$$(A+B)^\dagger = A^\dagger + B^\dagger$$

$$(AB)^\dagger = B^\dagger A^\dagger$$

$$\cancel{(AB)^\dagger |\psi\rangle} = \cancel{[(AB)^*]^T |\psi\rangle} = \cancel{(A^* B^*)^T |\psi\rangle}$$

$$AB|\phi\rangle = |\psi\rangle \Rightarrow \langle\phi|(AB)^\dagger = \langle\psi|$$

$$B|\phi\rangle = |\chi\rangle \Rightarrow \langle\phi| B^\dagger = \langle\chi|$$

$$\cancel{A|\chi\rangle} =$$

$$A(B|\phi\rangle) = A|\chi\rangle = |\psi\rangle \Rightarrow (\langle\phi|B^\dagger)A^\dagger = \langle\chi|A^\dagger = \langle\psi|$$

$$= \langle\phi|B^\dagger A^\dagger = \langle\psi|$$

$$(AB)^\dagger = B^\dagger A^\dagger$$

The matrix element of operator A in basis of $|u_m\rangle |u_n\rangle$

$n = 1, 2 \ldots$, $m = 1, 2 \ldots$

$$A_{mn} = \langle u_m | A | u_n \rangle =$$

Qn $A|\phi_n\rangle = \sqrt{n}\,|\phi_{n+1}\rangle$ $\quad n = 1, 2, \ldots$

Write down a matrix in basis of $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle \dots |\phi_n\rangle$

$\langle \phi_m|\phi_n\rangle = \delta_{mn}$

$A|\phi_1\rangle = 1|\phi_2\rangle \Rightarrow A_{11} = \langle\phi_1|A|\phi_1\rangle = \langle\phi_1|\phi_2\rangle = 0$

$A_{12} = \langle\phi_1|A|\phi_2\rangle = \sqrt{2}\langle\phi_1|\phi_3\rangle = 0$

$A_{13} = \langle\phi_1|A|\phi_3\rangle = \sqrt{3}\langle\phi_1|\phi_4\rangle = 0$

$A_{21} = \langle\phi_2|A|\phi_1\rangle = \langle\phi_2|\phi_2\rangle = 1$; $A_{31} = 0$

$A_{22} = 0$, A , $A_{32} = \sqrt{2}$

$A_{23} = 0$, $A_{33} = 0$

$$\begin{pmatrix} 0 & 0 & 0 \\ \sqrt{1} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

$A_{mn} = \langle\phi_m|A|\phi_n\rangle$

$= \sqrt{n}\langle\phi_m|\phi_{n+1}\rangle$

$= \sqrt{n}\,\delta_{m,n+1}$

A	$\|\phi_1\rangle$	$\|\phi_2\rangle$	$\|\phi_3\rangle$
$\langle\phi_1\|$	0	0	0
$\langle\phi_2\|$	$\sqrt{1}$	0	0
$\langle\phi_3\|$	0	$\sqrt{2}$	0

Que. $A|u_n\rangle = \sqrt{n+1}\,|u_{n-2}\rangle$

and $A|u_0\rangle = 0$ $\quad n = 0, 1, 2 \dots$

$A|u_1\rangle = 0$

$A|u_2\rangle = \sqrt{3}|u_0\rangle$

Then write down 4x4 matrix

A	A	$\|u_0\rangle$	$\|u_1\rangle$	$\|u_2\rangle$	$\|u_3\rangle$
$\langle\phi_0\|$	$\langle u_0\|$	0	0	$\sqrt{3}$	0
$\langle\phi$	$\langle u_1\|$	0	0	0	$\sqrt{4}$
	$\langle u_2\|$				
	$\langle u_3\|$				

$A_{nm} = \langle u_n|A|u_m\rangle$

$= \sqrt{m+1}\langle u_n|u_{m-2}\rangle$

$= \sqrt{m+1}\,\delta_{n,m-2}$

Mathematical Operator :-

$$D_x \phi(x) = \frac{\partial}{\partial x}\phi(x)$$

$$D_x^2 \phi(x) = D_x \cdot D_x \phi(x) = \frac{\partial^2}{\partial x^2}\phi(x)$$

In general $AB \neq BA$

$$[A,B] = AB - BA$$

Rules for Commutator Bracket :-

$$[A+B, C] = [A,C] + [B,C]$$
$$[A, f(A)] = 0$$
$$[AB, C] = A[B,C] + [A,C]B$$
$$[A, BC] = B[A,C] + [A,B]C$$
$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$$
$$[A, C] = 0$$

$$X f(x) = x f(x)$$
$$P_x f(x) = -i\hbar \frac{\partial}{\partial x} f(x)$$

$$[x, P_x] \Rightarrow [x, P_x] f(x) = (X P_x - P_x X) f(x)$$
$$= X P_x f(x) - P_x X f(x)$$
$$= X\left(-i\hbar \frac{\partial}{\partial x} f(x)\right) - \left(-i\hbar \frac{\partial}{\partial x} f(x)\right) x f(x)$$
$$= -i\hbar x \cancel{\frac{\partial}{\partial x}} f(x) + i\hbar \frac{\cancel{\partial}}{\cancel{\partial x}} f(x) + i\hbar x \frac{\partial f(\cancel{x})}{\cancel{\partial x}}$$
$$= i\hbar f(x)$$

$$\boxed{[x, P_x] = i\hbar}$$

<u>Qu</u> Using the relationship $[x, P_x] = i\hbar$ Find the value of $[x^2, P_x]$, $[x, P_x^2]$ & $[x^2, P_x^2]$

<u>Sol</u>

$$[x^2, P_x] = x[x, P_x] + [x, P_x]x$$
$$= i\hbar x + i\hbar x = 2i\hbar x$$
$$[x, P_x^2] = P_x[x, P_x] + [x, P_x]P_x$$
$$= 2i\hbar P_x$$
$$[x^2, P_x^2] = x[x, P_x^2] + [x, P_x^2]x$$
$$= xP_x[x, P_x] + x[x, P_x]P_x + P_x[x, P_x]x + [x, P_x]P_x x$$
$$= i\hbar[xP_x + xP_x + P_x x + P_x x]$$
$$= 2i\hbar[xP_x + P_x x]$$

Eigen <u>Values</u> :-

$$\boxed{A|\phi_n\rangle = a_n|\phi_n\rangle}$$

This is eigen value eqn ϕ_n is some scalar fun a_n is eigen value of A & corresponding eigen vector ϕ_n.

$$A|\phi_n\rangle - a_n|\phi_n\rangle = 0$$

$$(A - a_n I)|\phi_n\rangle = 0 \qquad |\phi_n\rangle \neq 0$$
$$|A - a_n I| = 0$$
↳ scalar eqn

When a_n will be non-repeted then it is said to be non-degenerate eigen value and if a_n is repeted then it is said to be degenerate eigen value.

If eigen values are non-degenerate then one can find three orthonormal eigen vectors. They can be uniquely defined. but when eigen values are degenerate then eigen vectors may or maynot be orthogonal ~~but~~ so they are not uniquely defined.

But in quantum mechanics one should always find the eigen vectors must be orthonormal.

EX-

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$|A - a_n I| = 0$$

$$\begin{vmatrix} 2-a_n & 0 & 0 \\ 0 & -a_n & 1 \\ 0 & 1 & -a_n \end{vmatrix} = 0$$

$$(2-a_n)(a_n^2-1) = 0 \Rightarrow a_n = 2, \pm 1$$

$$a_1 = 2, \quad a_2 = 1, \quad a_3 = -1$$

For $a_1 = 2$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 = k$ (let)

$-2x_2 + x_3 = 0 \Rightarrow x_3 = 2x_2$

$x_2 - 2x_3 = 0 \Rightarrow x_2 - 4x_2 = 0$

$x_2 = 0, \quad x_1 = 0$

$$\begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix}$$

For $a_2 = 1$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

let $x_2 = k_1$

$x_1 = 0$

$x_2 = x_3$ $\begin{pmatrix} 0 \\ k_1 \\ k_1 \end{pmatrix}$

For $a_3 = -1$

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ k \\ -k \end{pmatrix}$$

Eigen vectors. $\begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ k_1 \\ k_1 \end{pmatrix}, \begin{pmatrix} 0 \\ k_1 \\ -k_1 \end{pmatrix}$

$A|\phi_1\rangle = a_1|\phi_1\rangle$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = 2\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$2c_1 = 2c_1$

$\left.\begin{array}{l} c_3 = 2c_2 \\ c_2 = 2c_3 \end{array}\right]$ ← Not possible but possible only if $c_2 = c_3 = 0$

eigen vector $\begin{pmatrix} c_1 \\ 0 \\ 0 \end{pmatrix}$ so normalized eigen vector $\langle\phi_1|\phi_1\rangle = 1$

$|c_1|^2 = 1$

so $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $c_1 = 1$

$\langle\phi_2|\phi_2\rangle = 1$ $= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$2|k_1|^2 = 1 \Rightarrow k_1 = 1/\sqrt{2}$

$\langle\phi_3|\phi_3\rangle = 1$ eigen vector $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$$\langle \phi_1 | \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ k_1 \\ k_1 \end{pmatrix} = 0$$

$$\langle \phi_2 | \phi_3 \rangle = \frac{1}{2} \begin{pmatrix} 0 & k_1 & k_1 \end{pmatrix} \begin{pmatrix} 0 \\ k_1 \\ -k_1 \end{pmatrix} = 0$$

So any value of k and k_1 $|\phi_1\rangle, |\phi_2\rangle$ & $|\phi_3\rangle$ are orthonormal to each other.

$a_1 = 2$ $\quad |\phi_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\quad a_2 = 1$ $\quad |\phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$a_3 = -1$ $\quad |\phi_3\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$$a_1 |\phi_1\rangle + a_2 |\phi_2\rangle + a_3 |\phi_3\rangle = 0$$

$$a_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + a_2 \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} + a_3 \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = 0$$

$a_1 = 0$

$a_2 + a_3 = 0$ $\quad , \; a_2 - a_3 = 0$

$a_2 = a_3 = 0$

Ex = $\quad A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$$|A - a_n I| = 0$$

$$\begin{vmatrix} 1-a_n & 0 & 0 \\ 0 & -a_n & 1 \\ 0 & 1 & -a_n \end{vmatrix} = 0 \qquad (1-a_n)(a_n^2 - 1) = 0$$

$a_n = 1, \pm 1$