



IES/GATE

←————→

MECHANICAL ENGINEERING

VOLUME - 4

STRENGTH OF MATERIALS (SOM)



Index

SOM

| | |
|--|------------|
| 1. Mechanics of material | 1 |
| 2. Stress and Strain | 57 |
| 3. Principal stress and Strain | 112 |
| 4. Moment of Inertia and centroid | 138 |
| 5. Bending moment and shear force diagram | 151 |
| 6. Deflection of Beam | 178 |
| 7. Banding stress in Beam | 204 |
| 8. Shear stress in a Beam | 223 |
| 9. Torsion | 243 |
| 10. Thin Pressure vessel | 252 |
| 11. Thick pressure vessel | 257 |
| 12. Spring | 267 |
| 13. Theories of column | 272 |
| 14. Strain Energy method | 281 |
| 15. Theories of failure | 293 |

SOM

SOM

SOM / MOS / MOM / SM :-

Text book

- ⇒ Mechanics of material - Timoshenko and Gere
(CBS publication)
- ⇒ Mechanics of solid - Popov (PHI publication)
- ⇒ Mechanics of material Ramamida and Ashok Kumar Jain
(Laxmi publication)
- ⇒ Strength of Material - RAMAMRUTHAM
(DHANPAT RAI PUBLN)

IES - PAPER-II

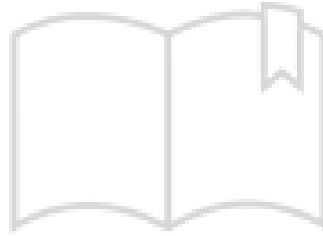
Objective - 65 to 70 marks.

conventional - 65 to 70

TOPIC OF ENGINE MECHANICS

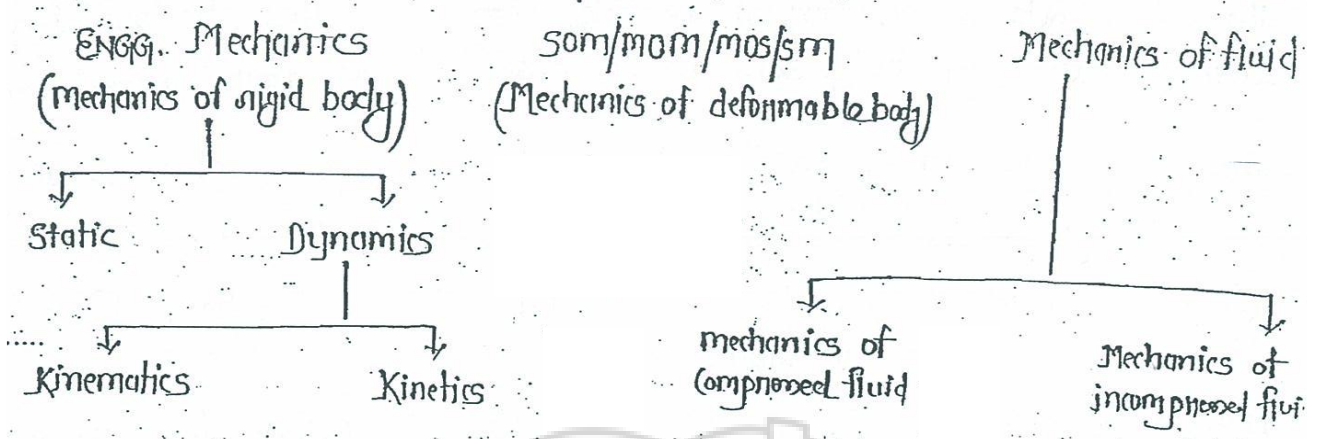
F.B.D.

- Resolution of force
- static equation cond^{ns}
- Centroid calculation
- Moment of Inertia
- Types of Support
- Support Reaction calculation



Chapter :- 1 Mechanics Of Material

MECHANICS

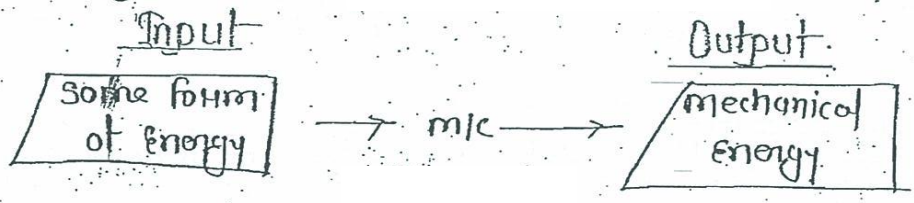


Mechanics is the branch of science which deals with the study of forces (both external and internal force) and their effect on the structure, machine, fluids.

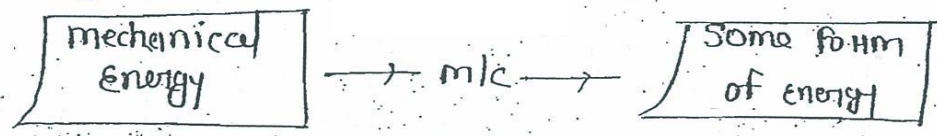
Machine :- It is combination of mechanism and mechanism is the combination of m/c element. It may be following type:-

1. Generating m/c.
2. Converting m/c.
3. Utilising m/c.

Generating Machine



Converting Machine



Utilising Machine :-

Receive the energy from the generating machine and utilise the same during their functionality.

shaft \rightarrow bearing \rightarrow key \rightarrow gear \rightarrow pulley \rightarrow belt \rightarrow fasteners etc. will be design element if there is only shaft.

When the Relative position of particle remains same by applying external force then body said to be displaced body (only displacement occur)

When Relative position of particle changes by applying external force then body said to be deformed body (deformation takes place)

Engine Mechanics :- Study of external force and their effect on rigid body.

Strength of Material :- It is study of internal resisting force developed due to elastic deformation of a body under the action of a load.



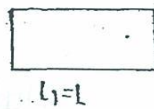
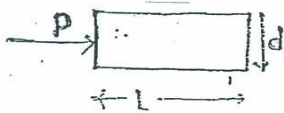
displacement



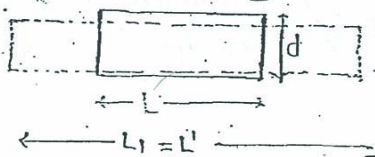
deformation (sub-deals with this condⁿ i.e. body under equilibrium)



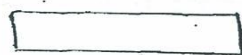
deformation and displacement (both occurs)



said to be displaced and called rigid body



body said to be deformed body.



deformed + displaced



All the component is assumed to under static equilibrium.

The aim of some subject is to derive the expression for the stresses, deformation, strain which are developed under different loading condition by using experimentally obtained elastic property like young modulus.

Design:- Ultimate aim of the design is to develop a drawing (ie. selection of an appropriate shape, selection of appropriate material, calculation of dimension by using some equation, selection of manufacturing process details, like surface finish, tolerance limit and fits) in such a way that the resulting machine component should perform its given functionality satisfactorily without any failure.

(design :- develop a part)

(fracture :- break in two component)
(failure :- when functionality do not fulfill)

Basic Requirement of m/c Element :-

1. High strength
2. High Rigidity
3. Cost should be low
4. High service life
5. low weight

Design Criterion :-

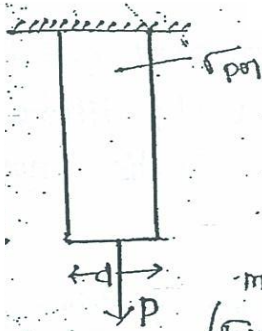
1. Strength criterion → based on permissible stress

not a property
 ↓
 obtained by taking safety of factor.
 Why use:- To find out the dimension of member

(a) Allowable (b) Working (c) Safe (d) design stress

2. Rigidity → based on permissible deformation

↳ (obtained from standard codes)



Condition for safe design w.r.t strength Criterion

max. stress induced \leq permissible stress

max. strength obtained by using some of
 $(\sigma_{max} = \frac{P}{A}, \tau_s = \frac{M}{Z}, \tau_s = \frac{T}{Z_p})$

↓
 (obtained by taking factor of safety)

$\frac{P}{A} \leq \frac{\text{failure stress}}{F.O.S.}$ OR σ_{per}

$\frac{4P}{\pi d^2} \leq \frac{\text{failure stress}}{N}$ OR $\frac{S_{yt}}{N}$

S = strength
 Y = yield
 T = tension

$$d \geq \sqrt{\frac{4PN}{\pi S_{yt}}}$$

$\Rightarrow d \propto \sqrt{N}$

$d \geq 45.2 \text{ mm}$ (assume) then means

Condition for safe design w.r.t Rigidity Criterion:

max. deformation induced \leq Permissible deformation

$(\delta_{max} = \frac{PL}{AE}, \theta = \frac{TL}{GJ}, \gamma = \frac{WL^3}{3EI}, \frac{WL^3}{48EI})$

$\frac{PL}{AE} \leq \delta_{per}$

$$d \geq \sqrt{\frac{4PL}{\pi E \delta_{per}}}$$

$d \geq 521 \text{ mm}$ (assume) then means

Assumption made in strength of Material Eqⁿ:

- Material is assumed to be both homogeneous and isotropic.
- Material obeys the hook's law.
- Component is assumed to be prismatic.
- Load is assumed as static load.
- Effect of self weight is neglected.

Static load :- When mag. and dirⁿ const wth time.

Prismatic :- All dimension are const or same throughout the struct.

Homogeneous :- A material is said to be homogeneous when it exhibits same elastic property at any pt in a given direcⁿ (ie. elastic property are independent of pt)

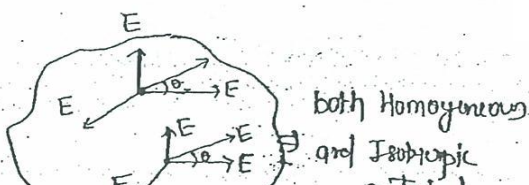
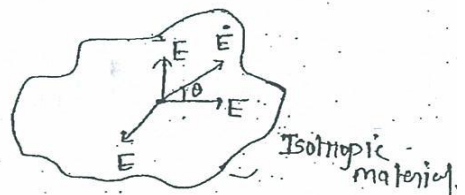
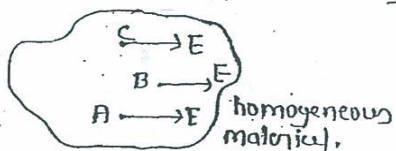
Isotropic :- A material is said to be isotropic when it exhibits same elastic properties in any direction at a given point. (ie. elastic property are independent of direction)

⇒ A material is said to be both homogeneous and isotropic when it exhibits same elastic property at any point in any diⁿ

⇒ Every homogeneous material need not be isotropic and vice-versa but few material are both homogeneous and isotropic

⇒ A material is said to be anisotropic when it exhibits dirⁿ dependent property of a point.

⇒ A material is said to be orthotropic when it exhibits diff. elastic property in orthogonal direction of a given point.
 ex: Any layered material like plywood.



Margine of safety (m):-

$$m = N - 1$$

and if $N = 1$

$$\text{then } m = 0$$

\Rightarrow failure design

and if $N > 1$

$$m > 0$$

\Rightarrow safe design

let

$$S_{ut} = 200 \text{ mpa}$$

(a) $N = 2$

$$\text{then } \sigma_{per} = \frac{200}{2} = 100$$

$$\text{and Reserve strength} = S_{ut} - \sigma_{pr} = 200 - 100 = 100$$

(b) $N = 4$

$$\text{then } \sigma_{pr} = \frac{200}{4} = 50$$

$$\text{and Reserve strength} = S_{ut} - \sigma_{pr} = 200 - 50 = 150$$

RESERVE STRENGTH:

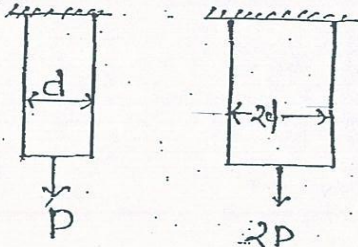
$$R.S. = \text{failure stress} - \text{Permissible stress}$$

Factor of safety is incorporated in design of a m/c component to ensure some amount of reserve strength in component in case of an accident (ie. in presence of unknown loading and unknown environment condⁿ, imperfect-workmanship, unreality of assumption made in some expⁿ).

In presence of factor of safety the stress induced in component may exceed permissible stress but lies below the failure stress.

$$\sigma_{per} < \sigma_{ind} < \text{failure stress}$$

Q. What is the factor of safety when dia of an axially loaded member and axial load both doubles.



1st case:- $\sigma_{ind})_1 = \frac{P_1}{A_1} = \frac{P}{A}$

$\sigma_{ind})_2 = \frac{P_2}{A_2} = \frac{2P}{4A} = \frac{1}{2} \left[\frac{P}{A} \right]$

as material same hence failure stress is same

$\sigma_{ind} \leq \text{failure stress}$

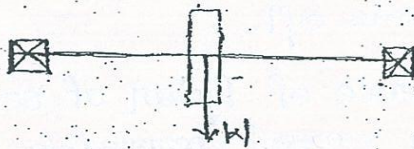
$\sigma_{ind} \propto \frac{1}{H}$

$\frac{\sigma_{ind})_1}{\sigma_{ind})_2} = \frac{H_2}{H_1}$

$2 = \frac{H_2}{H_1}$

$H_2 = 2H_1$

QAD:- Load is defined as an external force or a couple to which a component is subjected during its functionality. weight of a component w.r.t. another component. Centrifugal forces, inertia forces, gas load, belt tensions, twisting couple, wind forces, bending couples etc.



→ In the design of shaft

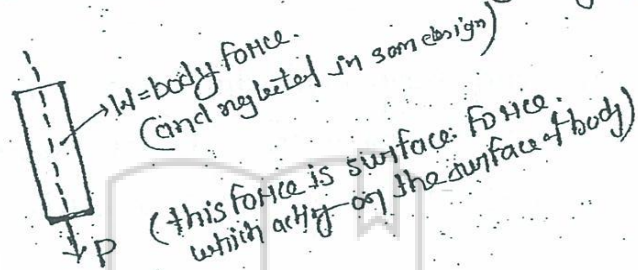
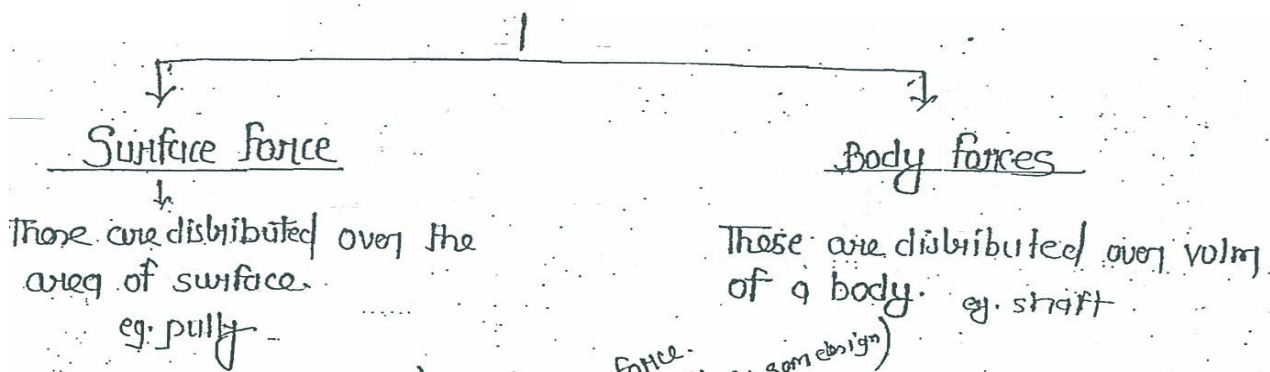
weight of pulley (W) is considered

weight of shaft is neglected.

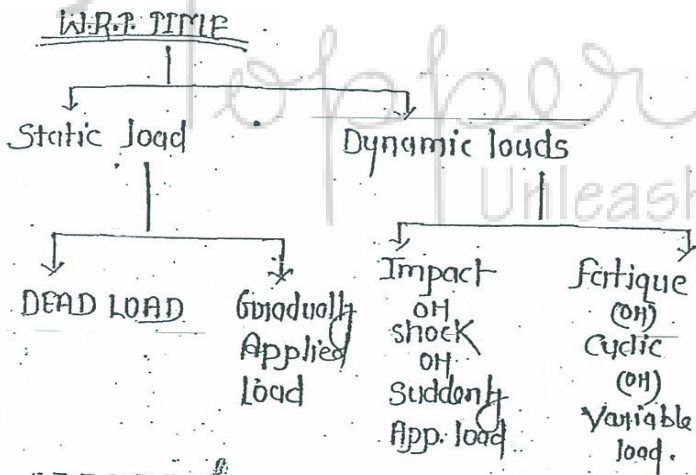
→ In design of bearing

weight of pulley and weight of shaft considered

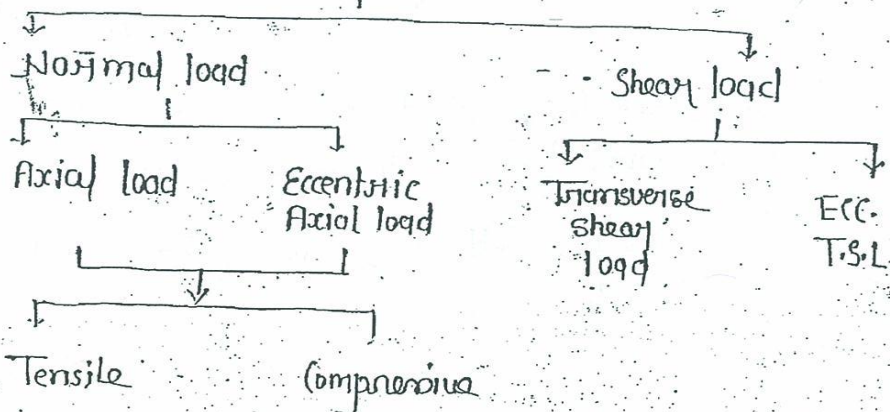
forces



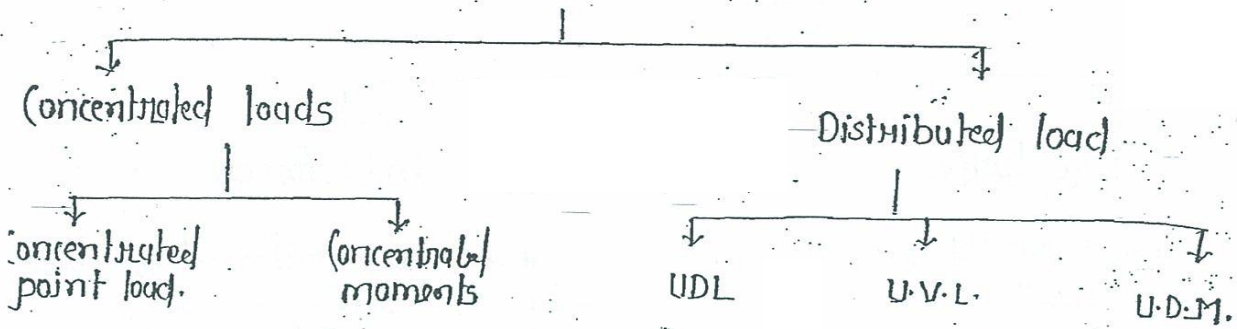
LOAD



Based on dirⁿ of load wth cross-section



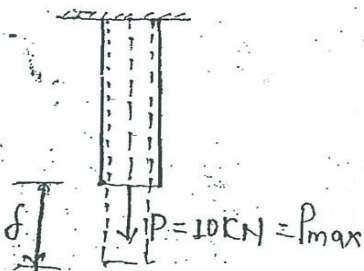
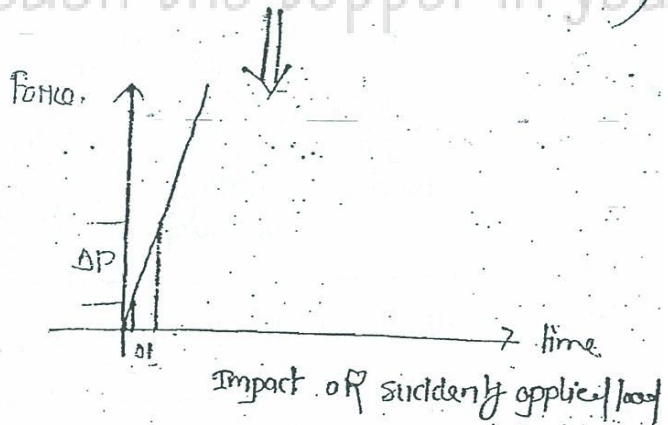
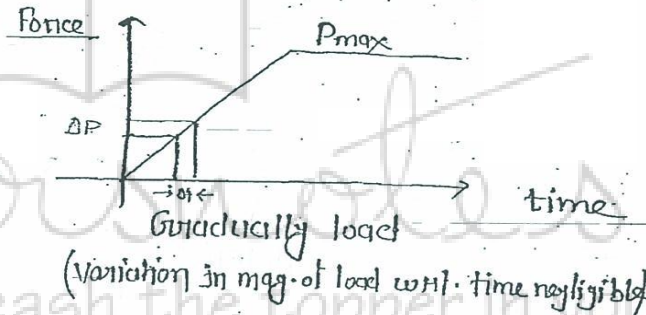
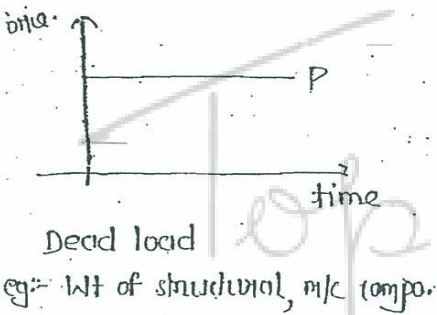
W.R.P. Distribution of load :-



static load :- When load mag. and dirⁿ are remains const.

dynamic load :- When load mag. and dirⁿ are continuously changes.

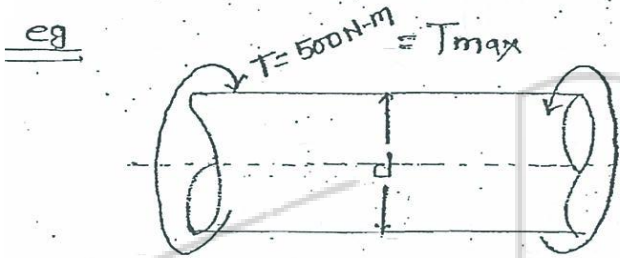
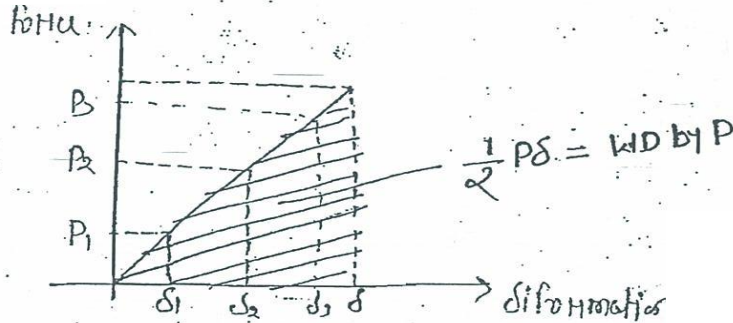
A.L :- When load gradually increases with time (mag. varies 0 → max)



EVERY gradually applied load is represented on component by its maximum magnitude.

(work done is considered as avg. work done in the entire span)

$$\boxed{\text{Work done by } P = \frac{1}{2} P \delta}$$



$$\text{WD by 'T'} = \frac{1}{2} T \theta$$

IMPACT LOAD: Impact load is a load which is acting for a short interval of time.

Impact is always fn of velocity

(Hence to overcome this difficulty we use helical gears at high velocity).
 eg. piston of ic engine, connecting rod, punching appⁿ, spring used in shock absorber, spur gear tooth under high pitch line velocity.

$$\sigma_{\text{impact}} = \sigma_{\text{static}} \times \text{I.F.}$$

$$\tau_{\text{impact}} = \tau_{\text{static}} \times \text{I.F.}$$

$$\text{I.F.} = 1 + \sqrt{1 + \frac{2h}{\sigma_{\text{static}}}}$$

σ_{static} and τ_{static} by using Som's Eqⁿ

$$h = \frac{v^2}{2g}$$

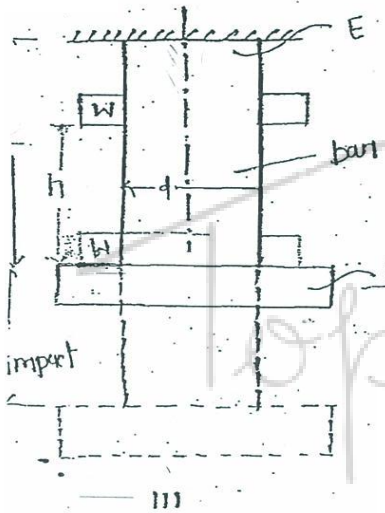
$I.F. \gg 2$

$\sigma_{\text{impact}} \gg 2 \sigma_{\text{static}}$

$\delta_{\text{impact}} \gg 2 \delta_{\text{static}}$

$h \rightarrow 0 \Rightarrow I.F. = 2$

(Impact load is known as suddenly applied load or instantaneous load)



① firstly Assume the given load is the static load.

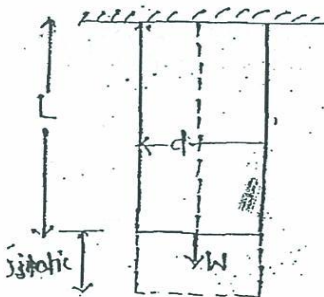
② $\sigma_{\text{static}} = \frac{WL}{AE}$ ——— ①

③ Impact factor
 $= 1 + \sqrt{1 + \frac{2h}{\delta_{\text{static}}}}$ ——— ②

④ $\sigma_{\text{static}} = \frac{W}{A}$ ——— ③

⑤ $\sigma_{\text{impact}} = \sigma_{\text{static}} \times I.F.$ ——— ④
 $= \dots \dots \dots \text{mpa}$

⑥ $\delta_{\text{impact}} = \delta_{\text{static}} \times I.F.$ ——— ⑤
 $= \dots \dots \dots \text{mpa}$



Work done by load (W) = Strain Energy of bar

$$W(h + \delta_I) = \frac{\sigma_I^2}{2E} \times A \times L$$

\downarrow
 $(\frac{\sigma_I L}{E})$

from above eqⁿ

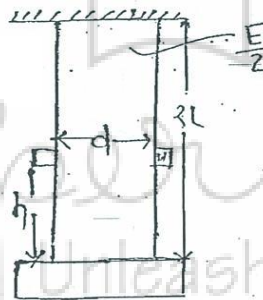
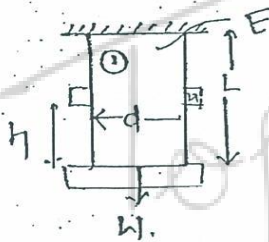
$$\Rightarrow \sigma_{st} \propto f(W \text{ and } A) \quad \text{i.e. } (\sigma_{static} \text{ is independent of } L \text{ and } E)$$

$$A \uparrow \Rightarrow \sigma_{st} \downarrow$$

$$\Rightarrow \sigma_{imp} \propto f[W, A, L, E, h]$$

$E \downarrow, A \uparrow, L \uparrow \Rightarrow$ strain energy of bar increases.
 $\Rightarrow \sigma_{impact} \downarrow$
 \Rightarrow chances of impact failure \downarrow

Both are subjected to Impact load and falling from same height



Static:-

$$\frac{(\sigma_{static})_1}{(\sigma_{static})_2} = 1 \quad \because W_1 = W_2$$

$$A_1 = A_2$$

Impact:-

$$\frac{(\sigma_{imp})_1}{(\sigma_{imp})_2} > 1 \quad \because L_2 > L_1$$

$$E_2 < E_1$$

$$\frac{(\sigma_{imp})_1}{(\sigma_{imp})_2} = \frac{(IF)_1}{(IF)_2} \quad \left[\because \sigma_{st}_1 = \sigma_{st}_2 = \frac{W}{A} \right]$$

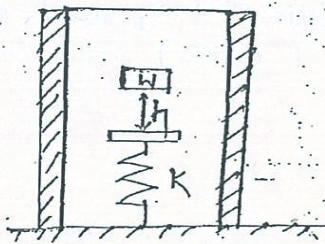
$$= 1 + \sqrt{1 + \frac{2 \times 100 \times 1000 \times 2500 \times 10^3}{5000 \times 5000}}$$

$$= 1 + \sqrt{1 + \frac{2 \times 100 \times 1000 \times 1000 \times 10^3}{5000 \times 10000}}$$

$$\approx 1.95$$

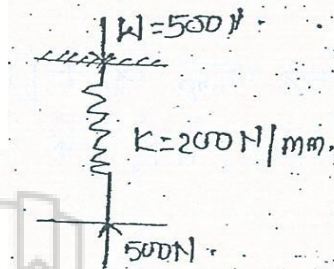
$L = 0.5m$
 $h = 0.1m$

For loading as shown in figure determine the maximum height the weight should allowed to fall, such that spring deflection should not exceed 5mm.



$W = 500\text{N}$
 $K = 200\text{N/mm}$ (under static)

(Rigidity criterion should use)



Assume given load is static

$$\delta_{\text{static}} = \frac{W}{K} = \frac{500}{200} = 2.5\text{mm}$$

$$\text{I.F.} = 1 + \sqrt{1 + \frac{2h}{\delta_{\text{static}}}} = 1 + \sqrt{1 + \frac{2h}{2.5}}$$

(iii) Condⁿ for safe,

$$\delta_{\text{impact}} \leq \delta_{\text{per}}$$

$$\delta_{\text{static}} \times \text{I.F.} \leq 5$$

$$2.5 \left[1 + \sqrt{1 + \frac{2h}{2.5}} \right] \leq 5$$

$$h \leq 10\text{mm}$$

Suddenly applied load

let $\delta_{\text{per}} = 10$

$$\delta_{\text{mp}} \leq \delta_{\text{per}}$$

$$\delta_{\text{static}} \times \text{I.F.} \leq 10$$

$$2.5 \left[1 + \sqrt{1 + \frac{2h}{2.5}} \right] \leq 10$$

$$h \leq 10\text{mm}$$

Fatigue load :- Fatigue loads are those load whose magnitude or direction or both magnitude and direction changes with time and these loads are repeatedly applied with time.

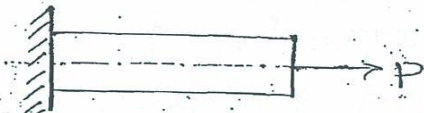


Fig:- Static load

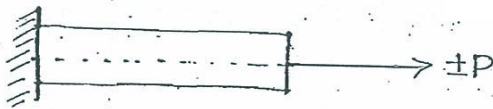


Fig:- (completely reversed fatigue load
(only direction of load changes)
mag. remains const)

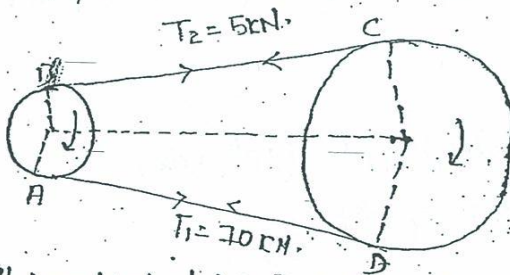


Fig:- fluctuating fatigue loads
(only mag. of load changes)



Fig:- Alternating fatigue loads
(both mag. and direction changes)

Example :-



Belt is subjected to fluctuating fatigue load.