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CBSE

CLASS-11th

THE CENTRAL BOARD OF SECONDARY EDUCATION

PHYSICS-II

SIERRA INNOVATIONS

CONTENT

PART - B

Chapter 9

MECHANICAL PROPERTY OF SOLIDS

Content

Elasticity

It is the property of a body, to regain its original size and shape when the applied force is removed. The deformation caused as a result of the applied force is called **elastic deformation.**

Plasticity

Some substance when applied force gets permanently deformed such materials are known as plastic material and this property is called plasticity.

Reason for Elastic behaviour of solids

When a solid is deformed, atoms move from its equilibrium position, when the deforming force is the interatomic force pulls the atom back to the equilibrium position hence they regain their shape and size. This can be better understood using the spring and ball structure on the left, where the ball represents the atoms and the force of spring represents the intermolecular force of attraction. When a ball is displaced from its original position by an external force the springs try to pull it back to original position when external force is removed. This is the reason for the elasticity of elastic material.

Stress And Strain

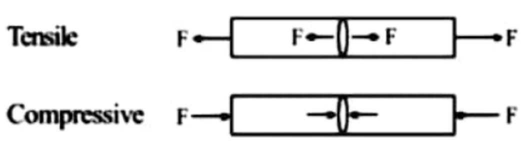
Stress

The restoring force per unit area is called the stress. If the force applied on a body is F and the area of cross section of the body is A then,

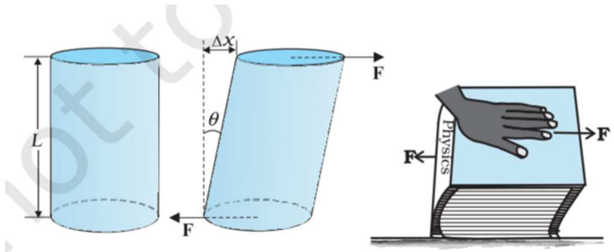
Magnitude of stress= F/A (N/m^2)

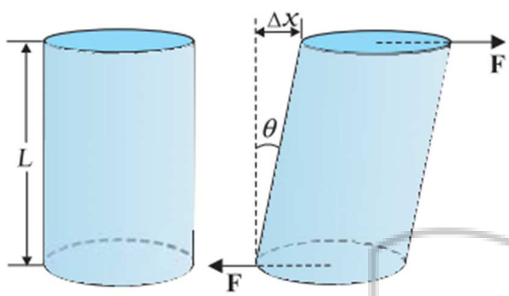
Types of stress

1. Tensile stress - when the two equal force is applied across the cross section of a body is known as tensile stress. Similarly when the forces compress the body it is known as compressive stress. Tensile and compressive stress can also be known as longitudinal stress.

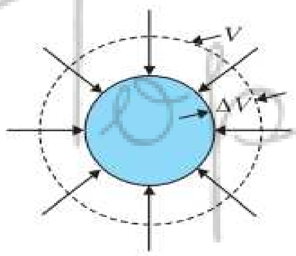


2. Shearing stress - The restoring unit force per unit area formed due to applying tangential force across the area of the cross section is called shearing stress





3. **Hydraulic stress -** When a body is compressed uniformly from all direction it exerts a internal restoring force this internal restoring force is known as hydraulic stress.



Strain

When a deforming force acts on a body, the body undergoes a change in it shape and size. The fractional change in configuration is called **strain**.

The formula is given by:

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Origonal dimension}} = \Delta x/x$$

Note:

1. $\text{Longitudnal Strain} = \frac{\text{Change in Lenght}}{\text{Origonal Lenght}}$

2. $\text{Volumetric Strain} = \frac{\text{Change in Volume}}{\text{Origonal Volume}}$

3.

$\text{Shearing Strain} = \frac{\text{Tangential Applied Force}}{\text{Area of Face}}$

Types of Strain

1. Longitudinal strain

It is the ratio of change in length of the body to original length.

Longitudinal strain = ΔL/L.

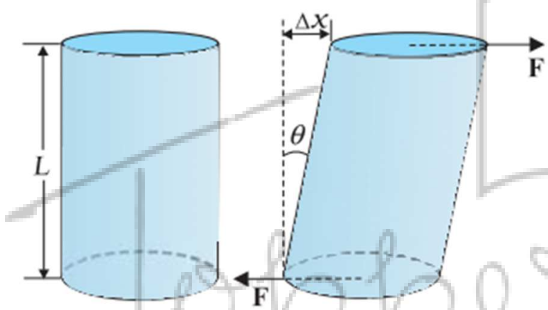
2. Lateral strain

It is the ratio of change of cross sectional diameter to the original diameter

Lateral stain = Δd/d

3. **Shearing strain**

It is the ratio of relative displacement of the faces to the length of the cylinder.



Shearing strain = Δx/L = tan ϴ

Since the angle of deformation is very small

tan ϴ≈ϴ

Where, ϴ is angular displacement of the body from the vertical position.

4. Volume strain

It is defined as change in volume to original **v**olume.

Volume strain = ΔV/V.

Hooke's Law

Hooke's law states that for small deformations the stress and strain are proportional to each other

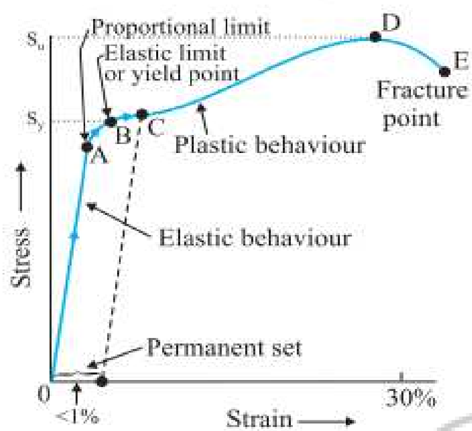
Stress ∝ strain

Stress = k ×strain

Where, k is the proportionality constant and is known as **modulus of elasticity.**

Stress-Strain Curve

The stress strain curve is a relation between the stress and strain of a given material. It helps to understand how a given material deforms under increasing load



This is how a typical graph of a material looks like. The graph varies with various materials. The graph has many regions in it

a. In the region **O to A** the curve is linear hence the material obeys Hooke's law in this region. So in this region the material is perfectly elastic, that is the body will regain its original dimensions when the applied force is removed.

b. In the region **A to B** stress and strain are not in linear relation, that is, stress is not proportional to strain, but still the material can regain its original shape and size once the force is removed.

c. The point B is known as **yield point/elastic limit,** the corresponding stress is known as **yield strength.**

d. When the stress developed exceeds the yield point the strain increases rapidly even for small changes in stress. The region from B to D shows this. If the load is removed at some point say C in between the B and D region the material will not regain its original dimensions. So even when the load is removed i.e. stress is zero strain is not zero. In such cases the material is said to have a **permanent set** and the deformation produced is said to be **plastic deformation.**

e. The point D in the graph is known as **ultimate tensile strength**. Beyond this point strain increases even if the load of force applied is decreased until material reaches fracture point E and brakes.

f. If the ultimate tensile strength D and breaking point E are close by then the material is said to be brittle and if they are far apart they are said to be ductile.

Elastomers

It is defined as the materials that can be stretched to cause large strains.

Eg:- tissue of aorta, rubber etc.

Elastic Moduli

Modulus of Elasticity

It is defined as the ratio of stress to strain and a characteristic property of materials.

1. **Young's Modulus**

   It is defined as the ratio of tensile stress to longitudinal strain

   It is denoted by the symbol "γ".

   $$\gamma = \sigma/\varepsilon$$

   From the previous equation for stress and strain we get

   γ = (F/A) / (ΔL/L)
   = (F x L) / (AxΔL)

   Since strain is unit less quantity unit of young's modulus is same as stress, N/m ^{2} or Pascal (Pa).

   Equation used to determine young's modulus

   $$\gamma = \frac{\sigma}{\varepsilon} = \frac{Mg}{\pi r^2} \cdot \frac{L}{\Delta L}$$

   Here, r = radius of cross section of the wire.
   M=mass that produced elongation.

   Therefore Mg is the force applied on the body, in this way we can generalize the above equation by replacing Mg by the deforming force applied.

2. **Shear Modulus**

   It is defined as the ratio of shear stress to its shear strain. It is denoted by the symbol "G".

   G = shearing stress (σ_N)/shearing strain

   G = (F/A)/(Δx/L)
   = (F × L)/(A × Δx)

It can be also written as

G = (F/A)/θ

G = F/($A \times \theta$)

Where, Ө is angular displacement of the body from the vertical position.

3. **Bulk Modulus**

The ratio of hydraulic stress to the corresponding hydraulic strain is called **Bulk Modulus**. It is denoted by symbol *B.*

B = -p/ (ΔL/ L).

The negative sign indicates the fact that with an increase in pressure, a decrease in volume occurs. That is, if *p* is positive, Δ*V* is negative. Thus, for a system in equilibrium, the value of bulk modulus *B* is always positive. SI unit of bulk modulus is the same as that of pressure.

Note: The reciprocal of bulk modulus is called compressibility (k)
K = 1/B

Poisson's Ratio

It is defined as the "Within elastic limit, lateral strain is directly proportional to longitudinal strain". The ratio of lateral strain to longitudinal strain is known as Poisson's ratio.

Poisson's ratio = $(\Delta d \times L)/(\Delta L \times d)$

Application of Elastic Behaviour of Material

The elastic behaviour of materials plays an important role in everyday life. All engineering designs require precise knowledge of the elastic behaviour of materials, knowledge of the elastic behaviour of materials. For example, while designing a building, the structural design of the columns, beams and supports require knowledge of strength of materials used. Have you ever thought why the beams used in construction of bridges, as supports etc. have across-section of the type I? Why does a heap of sand or a hill have a pyramidal shape? Answers to these questions can be obtained from the study of structural engineering which is based on concepts developed here.

Columns are very common. In both the cases, the overcoming of the problem of bending of beam under a load is of prime importance. The beam should not bend too much or break. Let us consider the case of a beam loaded at the centre and supported near its ends as shown in Fig.1

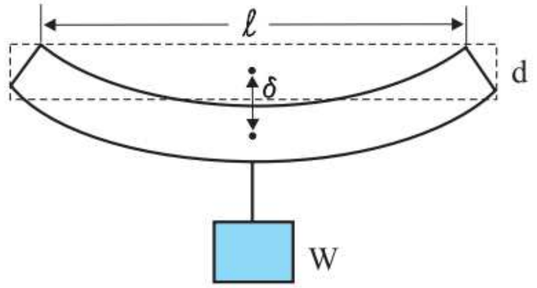


Fig.1 A beam supported at the ends and loaded at the center.

The length of bar is ℓ, breadth b and depth is d in which a load W is loaded at the center due to which the sags by amount, $\delta = W l^3 / (4bd^3Y)$.

For a given material, increasing the depth *d* rather than the breadth *b* is more effective in reducing the bending, since δ is proportional to d^{-3} and only to b^{-1}(of course the length *l* of the span should be as small as possible). But on increasing the depth, unless the load is exactly at the right place (difficult to arrange in a bridge with moving traffic), the deep bar may bend as shown in Fig.2 (b). This is called buckling. To avoid this, a common compromise is the cross-sectional shape shown in Fig. 2(c). This section provides a large load bearing surface and enough depth to prevent bending. This shape reduces the weight of the beam without sacrificing the strength and hence reduces the cost.

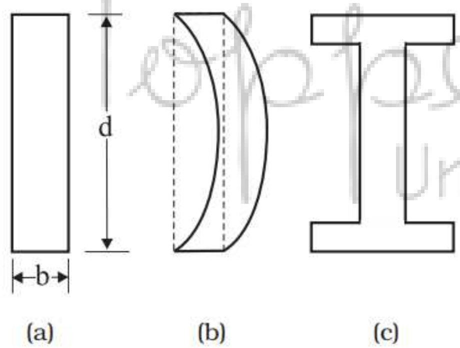


Fig.2 Different Cross-sectional shape of a beam

Some Important definitions

- **Elastic after effects:** The bodies return to their original state on the removal of the deforming force. Some bodies return to their original state immediately after the removal of the deforming force while some bodies take longer time to do so. The delay in regaining the original state by a body on the removal of the deforming force is called **elastic after effect**.
- **Elastic Fatigue:** The property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming force is called **elastic fatigue.**

- **Ductile Materials:** The materials which have large plastic range of extension are called **ductile materials**. Such materials undergo an irreversible increase in length before snapping. So, they can be drawn into thin wires. For e.g. copper, silver, iron, aluminium etc.
- **Brittle Material:** The material which has small range of plastic extension are called **brittle material**. Such material breaks very soon as stress is increased beyond their elastic limit, for e.g. cast iron, glass, ceramic.
- **Elastomers:** The material in which the strain is much larger than stress applied within the elastic limit is called **elastomers**, e.g. rubber.
- **Elastic Potential Energy of stretched wire:** When a wire is stretched, interatomic forces come into play which opposes the change. Work has to done against these restoring forces. The work done in stretching the wire is stored in it as its **elastic potential energy**.
- **Poisson Ratio:** When a deforming force is applied at the free end of a suspended wire of length $l$and diameter D, then its length increases by Δl but its diameter decreases by ΔD. Now two types of strains are produced by a single force.

Poisson Ratio = $$\frac{\text{Lateral Strain}}{\text{Longitudnal Strain}} = -\frac{l\Delta D}{D\Delta l}\ \frac{\frac{-\Delta D}{D}}{\frac{\Delta l}{l}} = -\frac{l\Delta D}{D\Delta l}$$

Note:

- **The negative sign shows the Lateral and Longitudinal strain in opposite sense. It is ratio of two strains, so it has unit less and dimensionless.**
- **The theoretical value of Poisson ratio lies between -1 and 0.5, but practical value lies between 0 to 0.5.**

Elastic Potential Energy In A Stretched Wire

When a wire is under tensile stress, work is done against the interatomic forces which tents to bring the body to its original dimension. This work is stored in the wire as a form of elastic potential energy.

Let a wire of length L and cross sectional area A is subjected to a deforming force F and let the elongation produced as a result of the force be l then,

F=YA x (l/L), Y is the young's modulus of the material

Now let there be a infinitesimal elongation dl

The work done for the infinitesimal elongation is

$$dW = F x dl$$

So, the amount of work done in elongating the wire from L to L+ l or l=0 to l=1 is

$$W = \int_0^l \frac{\gamma A l}{L} dl = \frac{\gamma A}{2} \times \frac{l^2}{L}$$

$$W = \frac{1}{2} \times \gamma \times (\frac{l}{L})^2 \quad \times AL$$

= $\frac{1}{2}$ Young's modulus $\times \epsilon^2 \times$ x volume of the wire

= $\frac{1}{2} \times$ stress x strain x volume of wire

This work is stored in the wire as elastic potential energy (U)
Therefore, the elastic potential energy per unit volume is (u)

$$u = \frac{1}{2} \times \sigma\varepsilon$$

Points to remember

- Stress is the resistance offered by the material against the deformation.

$$Stress = \frac{restoring\ force}{area}$$

- Strain is the fractional change in dimensions.

$$strain = \frac{change\ in\ dimension}{original\ dimension}$$

- Hooke’s law: Stress is directly proportional to strain within the elastic limit.

Stress α strain

$$E = \frac{stress}{strain}$$

Hooke’s law is valid only in the linear part of stress-strain curve.

- Young’s modulus: $Y = \frac{normal\ stress}{longitudinal\ strain} = \frac{F/A}{\Delta L/L}$

- Bulk modulus: $B = \frac{Hydraulic\ or\ bulk\ stress(volumetric)}{volumetric\ strain} = -\frac{F/A}{\Delta V/V}$

- Shear modulus: $G = \frac{shearing\ \ stress(tangential)}{shear\ strain} = \frac{F/A}{\Delta x/L}$

  Shear modulus is involved in solids only.

- Poisson's ration: $\sigma = \frac{Lateral\ strain}{longitudinal\ strain} = \frac{-\Delta r/r}{\Delta L/L}$

- The Young's modulus and shear modulus are relevant only for solids since only solids have lengths and shapes.
- Bulk modulus is relevant for solids, liquid and gases. It refers to the change in volume when every part of the body is under the uniform stress so that the shape of the body remains unchanged.
- Metals have larger values of Young's modulus than alloys and elastomers. A material with large value of Young's modulus requires a large force to produce small changes in its length.
- Stress is not a vector quantity since, unlike a force; the stress cannot be assigned a specific direction. Force acting on the portion of a body on a specified side of a section has a definite direction.

1. Mark Questions

Ques1. The stretching of a coil spring is determined by its shear modulus. Why?

Ans . When a coil spring is stretched, neither its length nor its volume changes, there is only the change in its shape. Therefore, stretching of coil spring is determined by shear modulus.

Ques2. The spherical ball contracts in volume by 0.1% when subjected to a uniform normal pressure of 100 atmospheres calculate the bulk modulus of material of ball?

Ans. Volumetric strain =ΔV/V.

$$=0.1\% = \frac{0.1}{100} = 10^{-3}$$

Normal stress =100 atmosphere= 100x10^5= $10^7 N$

So, bulk modulus of the ball

$$\frac{normal\ stress}{volumetric\ strain} = 10^7 / 10^{-3} = 10^{10} N$$

Ques3. Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

Ans. Hydraulic pressure exerted on slab, P=10 atm = $10 \times 1.103 \times 10^5 \mathrm{N/m^2}$

Bulk modulus of glass, B = $37 \times 10^9 Nm^{-2}$

Bulk modulus, B = $-P/(\Delta V/V)$

On substituting values, $\frac{\Delta V}{V}$ =2.73×10^{-5}

Hence, the fractional change in the volume of the glass slab is = 2.73×10^{-5}

Ques4. State Hooke's law.

Ans. Hooke's law states that the extension produced in the wire is directly proportional to the load applied within the elastic limit i.e. Acc to Hooke's law

$$\text{Stress} \propto \text{strain}$$

$$\text{Stress} = k \times \text{strain}$$

Where, k is the modulus of elasticity.

Ques5. What are ductile and brittle materials?

Ans. Ductile materials are those materials which show large plastic range beyond the elastic limit. eg:- copper, Iron Brittle materials are those materials which show very small plastic range beyond the elastic limit. eg:- Cast Iron, Glass.

Ques6. Define rigid body?

Ans. A rigid body is a solid body which has zero deformation or so small that can be neglected. The distance between any two given points on a rigid body remains constant in time regardless of external forces exerted on it. A rigid body is usually considered as a continuous distribution of mass.

Ques7. What is Modulus of rigidity of ideal liquids?
Ans. Since the ideal fluid is incompressible so, the modulus of rigidity of ideal fluid is **zero**.

Ques8. A spring is stretched by applying a load to its free end. The strain produced in the spring is?
Ans. longitudinal and shear.

Ques9. Is stress a vector quantity?
Ans. No, because stress is a scalar quantity.

$$\text{Stress} = \frac{\text{Magnitude of internal Reaction Force}}{\text{Area of Cross Section}}$$

Ques10. A thick wire is suspended from a rigid support, but no load is attached to its free end. Is this wire being under stress?
Ans. Yes, the wire is under stress due to its own weight.

Ques11. What is Young Modulus for perfect rigid body?
Ans. Young's Modulus = $\frac{F}{A} \times \frac{L}{\Delta L}$

For Perfectly Rigid body, $\Delta L = 0$
Hence, Young's Modulus = ∞

Ques12. The temperature of a wire is doubled. The Young's modulus of elasticity?
Ans. The Young's modulus is inversely proportional to the change in length (ΔL). So, when a wire is heated then the length of wire is also increased and distance between atoms also increased. Hence, The Young's Modulus will decrease.

Ques13. Shear Modulus of a material is generally less than Young's Modulus, what does it signifies?
Ans. As we know that it is easier to slide layer of atoms of solids over one another (Young's Modulus) than pull them apart or to squeeze them to close together (Shear Modulus).

Ques14. Water is more elastic than air. Why?
Ans. Since volume elasticity is the reciprocal of compressibility and since air is more compressible than water hence water is more elastic than air.

2. Marks Questions

Ques1. Stress and pressure both are force per unit area, then, in what respect the stress is different from pressure?

Ans. Pressure is external force per unit area while stress is the internal restoring force which comes into play in deforming body acting transversely per unit area of a body. Another difference is Pressure is a vector quantity while Stress is scalar.

Ques2. A steel cable with a radius of 1.5 cm supports a chair lift at ski area. If the maximum stress is not exceeded 10^8 N/m^2, then what is the maximum load the cable can support?

Ans. Given, the radius r of steel cable = 1.5 cm = 1.5 x 10^{-2} m

Maximum stress = 10^8 N/m^2

Area of cross section A = πr^2 = 3.14 x (1.5×10^{-2}) m^2

= 3.14 x 2.25 x 10^{-4} m^2

$$\text{Maximum Stress} = \frac{\text{Maximum Force}}{\text{Area of Cross Section}}$$

Or, Maximum force = Maximum stress x Area of Cross section

= 10^8 x (3.14 x 2.25 x 10^{-4}) N

= 7.065 x 10^4 N.

Ques3. A wire of length 7.5 m has a percentage strain of 0.024% under a tensile force. Determine the extension in wire?

Ans. The original length L = 7.5 m

$$\text{Strain} = \frac{\Delta L}{L} = 0.024\% = \frac{0.024}{100}$$

ΔL = Strain x L

Or, extension ΔL = (0.024/100) x L = $\frac{0.024}{100}$ x 7.5 = 1.8 x 10^{-3} m = 1.8 mm.

Ques4. Two wires A and B are of the same material. Their lengths are in the ratio 1:2 and the radiuses are in ratio 2:1. If they are pulled by same force, then what will be the ratio of their increased Length?

Ans. As we know ΔL = $\frac{FL}{AY}$, $\frac{L_A}{L_B} = \frac{1}{2}$ and $\frac{r_A}{r_B} = 2$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{L_A}{\pi r^2{}_A} \times \frac{\pi r^2{}_B}{L_B}$$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{L_A}{L_B} \times \frac{r^2_B}{r^2_A} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

Ques5. The stress-strain graphs for materials *A* and *B* are shown in Fig. a.

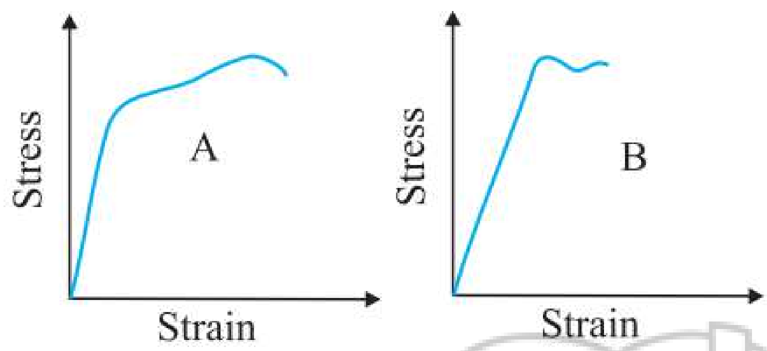


Fig. a

The graphs are drawn to the same scale.

(a) Which of the material has greater Young's Modulus?

(b) Which of two is stronger?

Ans. (a) The slope of graph A is more than B; hence the A has greater Young's Modulus than B.

(b). Material A is stronger than B because it can withstand more load without breaking.

Ques6. The Young's modulus for steel is much more than that for rubber. For the same longitudinal strain, which one will have greater tensile stress?

Ans. The Young Modulus is given by, $Young\ Modulus = \frac{Longitudnal\ stress}{longitudnal\ strain}$

For the same longitudinal strain Young Modulus ∝ Stress

Therefore, $\frac{Y_{steel}}{Y_{rubber}} = \frac{Stress_{steel}}{Stress_{rubber}}$

Given, $\frac{Y_{steel}}{Y_{rubber}} > 1$

Hence, $(Stress)_{steel} > (Stress)_{rubber}$

So, steel has more tensile strength than rubber.