Toppersnotes

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NEET - UG

NATIONAL TESTING AGENCY

Physics

Volume - 1

SIERRA INNOVATIONS

Contents

SYSTEM OF UNITS

Introduction:

CGS System:

* This system is based on **centimeter**, **gram** and **second** as the fundamental units of length, mass and time respectively. In this system, unit of force is dynes, unit of energy is ergs, and so on.

FPS System:

* This system is based on **foot**, **pound** and **second** as the fundamental units of length, mass and time respectively. In this system, unit of force is poundal, unit of energy is foot-poundal and so on.

MKS System:

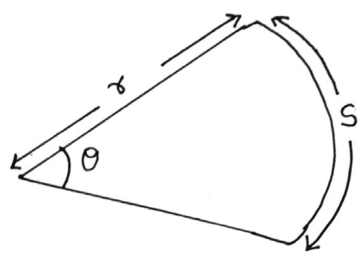
* This system is based on **metre**, **kilogram** and **second** as the fundamental units of length, mass and time respectively. In this system, unit of force is Newton, unit of energy is Joule and so on.

International System of Units (S.I.):

* The general conference of weights and measures held in 1971 decided a new system of units which is known as the International System of Units. It is abbreviated as S.I from the French name Le Systeme International d' Unites. It is based on the seven fundamental units.

2 supplementry Units:

* Plane angle
* Angle formed by an arc at a point.



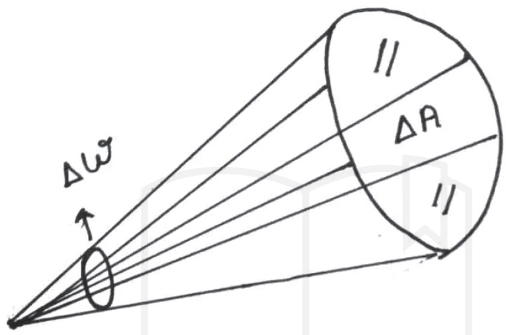
$$\text{Angle} = \frac{Arc}{radius} = \left(\frac{s}{r}\right)$$

= Dimensionless

= Unit – Radian

Solid Angle:

Angle formed by an area at a point—



$$\Delta\omega = \frac{\Delta A}{r^2}$$

Unit – Staradian

Dimensions:

* We know that derived units of all physical quantities can be obtained from the seven fundamental units and two supplementary units. Thus representing mass by (M), length by (L), time by (T), electric current by (A), temperature by (K), etc., all physical quantities can be expressed in terms of (M), (L), (T), (A), (K), etc.
* The dimensions of a physical quantity are the powers to which the fundamental quantities must be raised to represent the given physical quantity.

For Example:

$$[\text{Density}] = \frac{[Mass]}{[Volume]} = \frac{[M]}{[L^3]} = [ML^{-3}]$$

or $[M^1L^{-3}T^0]$

So, the dimensions of density are 1 in mass, - 3 in length and 0 in time. The dimensional formula of density is thus represented as $[ML^{-3}]$ or $[ML^{-3}T^0]$

The constants such as π, 1/2 or trigonometric functions such as $\sin\theta$, etc. have no units and dimensions.

The following table gives the dimensional formulae and S.I. units of some physical quantities.

Dimensional Formulae of some physical quantities and their S.I. units.

S. No.	Physical Quantity	Formula Used	Dimension	S.I. Unit
1.	Area	Length × breadth	$[M^0L^2T^0]$	m^2
2.	Volume	Length × breadth × height	$[M^0L^3T^0]$	m^3
3.	Density	Mass/volume	$[ML^{-3}T^0]$	$kg\ m^{-3}$
4.	Velocity	Displacement/time	$[M^0LT^{-1}]$	ms^{-1}
5.	Acceleration	Velocity/time	$[M^0LT^{-2}]$	ms^{-2}
6.	Force	Mass × acceleration	$[MLT^{-2}]$	$kg\ ms^{-2}$ or N (Newton)
7.	Pressure	Force/area	$[ML^{-1}T^{-2}]$	$Kg\ m^{-1}s^{-2}$ or Pa (Pascal)
8.	Work	Force × displacement	$[ML^2T^{-2}]$	$kg\ m^2s^{-2}$ or (Joule)
9.	Potential Energy	Mgh	$[ML^2T^{-2}]$	$kg\ m^2s^{-2}$ or J (Joule)
10.	Kinetic Energy	$\left(\frac{1}{2}\right)mv^2$	$[ML^2T^{-2}]$	$kg\ m^2s^{-2}$ or J (Joule)
11.	Power	Work/time	$[ML^2T^{-3}]$	$kg\ m^2s^{-3}$ or W (Watt)
12.	Momentum	Mass × velocity	$[MLT^{-1}]$	$kg\ m\ s^{-1}$
13.	Angle	Arc/radius	$[M^0L^0T^0]$	rad
14.	Angular Velocity	Angle/time	$[M^0L^0T^{-1}]$	$rad\ s^{-1}$
15.	Impulse	Force × time	$[MLT^{-1}]$	$kg\ ms^{-1}$ or Ns
16.	Moment of Inertia	Mass × $(Distance)^2$	$[ML^2T^0]$	$kg\ m^2$
17.	Torque	Force × distance	$[ML^2T^{-2}]$	$kg\ m^2s^{-2}$ or N m
18.	Angular Momentum	Mass × Velocity × distance	$[ML^2T^{-1}]$	$kg\ m^2s^{-1}$
19.	Stress	Force/Area	$[ML^{-1}T^{-2}]$	$kg\ m^{-1}\ s^{-2}$ or $N\ m^{-2}$
20.	Strain	$\Delta L/L$ or $\Delta V/V$	$[M^0L^0T^0]$	No unit
21.	Modulus of Elasticity	Stress/strain	$[ML^{-1}T^{-2}]$	$kg\ m^{-1}\ s^{-2}$ or $N\ m^{-2}$
22.	Surface Tension	Force/length	$[ML^0T^{-2}]$	$kg\ s^{-2}$ or $N\ m^{-1}$
23.	Frequency	$(\text{Time Period})^{-1}$	$[M^0L^0T^{-1}]$	s^{-1} or Hz (Hertz)
24.	Planck's constant	Energy/frequency	$[ML^2T^{-1}]$	$kg\ m^2s^{-1}$ or Js
25.	Electric Charge	Current × time	$[M^0L^0TA]$	As or C (Coulomb)

26.	Potential Difference	Power/electric current	$[ML^2T^{-3}A^{-1}]$	$kg\ m^2s^{-3}\ A^{-1}$ or V (Volt)
27.	Resistance	Pot. Diff./electric current	$[ML^2T^{-3}A^{-2}]$	$kg\ m^2s^{-3}\ A^{-2}$ or Ω (ohm)
28.	Electric Dipole Moment	Electric charge × distance	$[M^0LTA]$	mAs or Cm
29.	Electric field	Force/electric charge	$[MLT^{-3}A^{-1}]$	$kg\ ms^{-3}A^{-1}$ or NC^{-1}
30.	Magnetic field	Force/(current × length)	$[MT^{-2}A^{-1}]$	$kg\ s^{-2}A^{-1}$ or T(Tesla)

* If a quantity is unique then its dimension will also be unique but reverse may or may not be true.

  Eq. $[ML^2T^{-2}]$ may represent many quantities like work done, Torque, Energy etc.

  Eq. $[MT^{-2}]$ – represent spring constant and surface tension and these do not represent similar physical quantitites.

* If a quantity is unitless, it must be dimensionless, but reverse may or may not be true.

  For ex. Relative density is unit less & also dimensionless but angle is dimensionless but still have unit Radian.

Rules Regarding Dimension:

1. Addition and subtraction between two quantities are possible if and only if quantities have similar unit or dimension.

   Example: $A \pm B$ is meaningful only if A and B have same dimension however same restriction is not in multiplication and division.
2. In case of $\sin x$, $\cos x$, $\tan x$ etc, x must be dimensionless.
3. $\log x$, $e^x \rightarrow x$ must be dimensionless

Ques.: $v = at + bt^2 + c + + \dfrac{d}{t+d}$, what are the dimensions of a, b, c, d?

$v \rightarrow$ Velocity, $t \rightarrow$ time, a, b, c, d = constant

Soln.: d must have dimension of time

$$d \rightarrow [T]$$

1. $$v = at$$
2. $$[LT^{-1}] = a[T]$$
   dim. of $$a = [LT^{-2}]$$
3. $$v = bt^2$$

$$[LT^{-1}] = b[T^2]$$

dim. of $b = [LT^{-3}]$

4. $$\frac{c}{t+d} = [LT^{-1}]$$

$$\frac{c}{[T]} = [LT^{-1}]$$

dim. of $c = [L]$

a : $[LT^{-2}]$
b : $[LT^{-3}]$
c : $[L]$
d : $[T]$

Ques.: If u is P.E., x is distance, t is time and are related as—

$$u = \frac{bx^2}{t^2 + c}$$

Find dimensions of b & C

Solns.: Dim. of $\boxed{c = [T^2]}$

$$[ML^2T^{-2}] = \frac{b[L^2]}{[T^2]}$$

Dim. of $\boxed{b = [M]}$

Ques.: $x = A \cdot \sin(Bt) + C \cdot \cos(Dx)$

If $x \rightarrow$ distance   Find dimension of A, B, C and D
If $t \rightarrow$ time

Solns.: $B \cdot t$ must be dimensionless

$$B = [T^{-1}]$$

$$D = [L^{-1}]$$ (Dx must be dimensionless)

$$[L] = A \sin(Bt)$$

Dim. of

A $\rightarrow$ [L] (sin Bt, is dimensionless)

dim of C $\rightarrow$ [L] (Cos Dx, is dimensionless)

Ques.: $$x = a \log\left[\frac{bt^2}{c+a}\right]$$

if $x \rightarrow$ distance
$t \rightarrow$ time

Then find dimension of a, b, c

Solns.: dimension of $C \to [L]$ (As C is added to x)

dimension of $b \to \frac{[L]}{[T^2]}$ (AS $(bt^2/c + x)$ will be dimensionless)

dimension of $b \to [LT^{-2}]$

dimension of $a = [L]$ as log of anything as dimension Less

$a = [L]$

$b = [LT^{-2}]$

$c = [L]$

Ques.: $F = at + bt^2$

$F \to$ Force, $t \to$ time, then find the dimension of a and b.

Solns.: Dimension of F = Dimension of at = Dimension of bt^2

$MLT^{-2} = a[T]$

Dimension of A

$a = [MLT^{-3}]$

$b[T^2] = MLT^{-2}$ (Dimension of force)

Dimension of $b = [MLT^{-4}]$

Ques.: $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

P : Pressure
V : Volume
R : Gas constant
T : Temperature

A, b constant. Find dimension of a and b.

Soln.: Dimension of P = Dimension of $\frac{a}{L^6}$

$\Rightarrow$ $a = [ML^5T^{-2}]$

Dimension of b = dimension of V

dimension of $b = [L^3]$

Application of Dimensions:

* To check whether the equation is dimensionally correct or not.

Step I: find dimension of LHS

Step II: find dimension of RHS

Step III: If dimension of LHS = Dimension of RHS

Then eq. will be dimensionally correct otherwise not.

For ex. check the equation.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

LHS

$$\text{Dimension of LHS} = [T]$$

RHS

$$\text{Dimension of RHS} = \text{dimension of } \sqrt{\frac{l}{g}}$$

$$= \sqrt{\frac{[L]}{[LT^{-2}]}}$$

$$= \sqrt{T^2}$$

$$= T$$

$$\text{Dimension}_{RHS} = \text{Dimension}_{LHS}$$

Hence equation is dimensionally correct.

Example: $H = \frac{2s\cos ,}{drg}$

where

$H \rightarrow$ Height
$s \rightarrow$ surface tension
$d \rightarrow$ density
$r \rightarrow$ radius
$g \rightarrow$ gravitational acceleration

Check whether the equation is dimensionally correct or not.

Soln.: Dimension of LHS $= [L]$

Dimension of RHS: Surface Tension $= [MT^{-2}]$

Dimension of Density $= ML^{-3}$

Dimension of gravitational accⁿ $= [LT^{-2}]$

Dimension of RHS $= \frac{[MT^{-2}]}{\frac{[M]}{[L^3]}[L][LT^{-2}]}$

Yes the equation is dimensionally correct

Note:

* If an equation is physically correct then it must be dimensionally correct but – reverse may or may not be true.

  For eg. $T = \sqrt{\frac{l}{g}}$ is dimensionally correct but physically incorrect.

  To change the system of Measurement

* Any physical quantity = n.u. – proper unit

  n. ↓ Numerical value

  Distance between Kanpur and Lucknow= 80 km

  = 80,000 M

  So, 80 & 80,000 are numerical value and km and m. are proper unit

* For any physical qty.

$$= \left[n \propto \frac{1}{u}\right]$$

$$n_1u_1 = n_2u_2$$

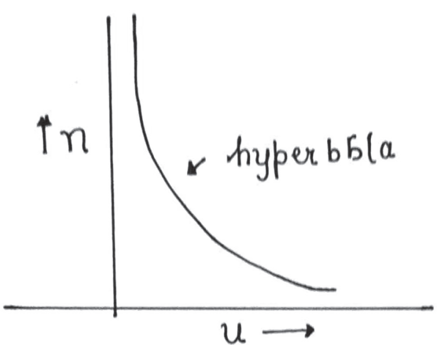
If $n_1 > n_2$

Then $U_1 < U_2$

n.u. = const.

Check above if we go from Km – m i.e towards Smaller unit, numerical Value Increases.

$$n \longrightarrow 0$$
$$u \longrightarrow \infty$$
$$n \longrightarrow \infty$$
$$u \longrightarrow 0$$



Graph b/w numerical value & its unit.

Ques.: Velocity of a train is 720 km/hr

Find its speed in $\frac{km}{min}$, $\frac{M}{S}$ and $\frac{M}{Min}$

Soln.: 1.

$$U_1 n_1 = U_2 n_2$$

$$v = 720 \frac{km}{hr} = x \frac{km}{min}$$

$$720 \frac{km}{60 \cdot min} = x \frac{km}{min}$$

$$x = 12$$

Ans.: 12 km/min.

2.

$$n_1 u_1 = n_2 u_2$$

$$v = 720 \frac{km}{hr} = x \cdot \frac{M}{S}$$

$$= x = 200$$

Ans.: 200 M/S

3.

$$n_1 u_1 = n_2 u_2$$

$$720 \frac{km}{hr} = x \cdot \frac{M}{Min.}$$

$$720 \cdot \frac{1000 \text{ M}}{60 \text{ Min.}} = \frac{x \cdot M}{Min.}$$

$$x = 1200$$

Ans.: 12000 m/min

MKS	CGS
Unit of force = 1 Newton	Unit of force = 1 Dyne
Unit of Energy = 1 J	Unit of energy = 1 erg s

Ques.: Find the relation between Newton & Dyne & also between Joule & Erg.

Solns.: Let

$$1 \text{ N} = x \cdot \text{dyne},$$

$$F = 1 \text{ N} = 1 \frac{kg \cdot m}{s^2} = \frac{x \, gm \, cm}{s^2}$$

$$\frac{1 \times 10^3 \, gm \times 100 \, cm}{s^2} = \frac{x \, gm \, cm}{s^2}$$

$$x = 10^5$$

$$1 \text{ N} = 10^5 \text{ dyne}$$

let

$$1 \text{ J} = x \text{ dyne}$$

$$* \quad E = 1\ J = \frac{1 kg\, m^2}{sec^2} = \frac{x\, gm\,(cm)^2}{sec^2}$$

$$1 \times 10^3 \frac{\cancel{gm} \times (100\ \cancel{cm})^2}{\cancel{sec}^2} = \frac{x\ \cancel{gm}\ \cancel{cm}^{\cancel{2}}}{\cancel{sec}^{\cancel{2}}}$$

$$10^3 \times 100 \times 100 = x$$

$$x = 10^7$$

$$1\ J = 10^7\ ergs$$

Ques.: There is a new kind of system called 'STAR'system which is defined as

$$1\ kg^* = 5\ kg$$
$$1\ M^* = 10\ M$$
$$1\ S^* = 20\ sec$$

Find the value of $1\ J$ in this new STAR system

Soln.:

$$1\ J^* = \frac{1 kg^* \cdot (1 m^*)^2}{(1 s^*)^2}$$

$$= \frac{5 kg \cdot (10 m)^2}{(20 s)^2}$$

$$= \frac{5 \times 100 J}{400} = \frac{5 J}{4}$$

$$1\ J = \frac{4}{5}\ J^*$$ **Ans.**

To Construct a Physical Equation:

Ques.: Time period of a simple pendulum depends upon mass of bob, length of string, acc^n due to gravity, find (a) suitable formula for time period of pendulum.

Soln.: Acc. to q:

$$T \propto M^a l^b g^c$$

$$T = k,\ m^a l^b g^c$$

$$[M^0 L^0 T^1] = [M]^a [L]^b [LT^{-2}]^c$$

$$[M^0 L^0 T^1] = [M]^a_0\ [L]^{b+c}_0\ [T]^{-2c}$$

Compare the power

$$a = 0$$
$$b + c = 0$$

$$-2c = 1$$

$$C = \frac{-1}{2}$$

$$b = -C$$

$$b = \frac{1}{2}$$

$$T \propto M^0 l^{1/2} g^{-1/2}$$

$$T = k.1\sqrt{\frac{l}{g}}$$

$$T = k\sqrt{\frac{l}{g}}$$

$k = 2\pi$ (By experiements)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Limitations of the Method:

1. We can't find formula which has addition or subtraction like $V = u\ tat.$
2. We can't find formula which has power function like $N = N_0 \cdot e^{-\lambda t}$
3. We can't find formula which has trigonometric or log function for ex. $x = A \sin (wt)$
   Sound level $= 10 \log (I/I_0)$

Que.: The heat produced in a wire depends upon the current, the resistance of the wire and the time. Dimension of resistance is $[ML^2T^{-3}A^{-2}]$ and heat is a form of energy, find a suitable formula for heating in a wire.

Soln.: Let
$$H \propto I^a \cdot R^b \cdot t^c$$

$\Rightarrow$
$$H \propto [A]^a, [ML^2T^{-3}A^2]^b\ {}_0[T]^c$$

$$H = [ML^2T^{-2}] = K[A]^{a-2b}\ [M]^b_0[L^2]^b, [T]^{c-3b}$$

From comparing

$\Rightarrow$ $b = 1$

$\Rightarrow$ $c - 3b = -2$

$$c = 3b - 2$$
$$= 3(1) - 2 = 1$$

$\Rightarrow$ $a - 2b = 0$

$$a = 2(1) = 2$$

$$H \propto I^a \cdot R^b \cdot t^c$$

$$\propto I^2Rt$$

$$H = K \cdot I^2\ Rt \text{ (from experiments } K = 1)$$

$$\Rightarrow \quad H = I^2\ Rt$$

Some More Examples:

Ques.: In the equation $\int \frac{dt}{\sqrt{2at - t^2}} = a^x \sin^{-1}\left(\frac{t}{a} - 1\right)$

The value of x is –

Solns.: LHS:

$$\sqrt{2at - t^2} \rightarrow [T^1]$$

$$dt \rightarrow [T^1]$$

hence, LHS $\rightarrow$ dimensionless

RHS: $\sin^{-1}\left(\frac{t}{a}\right) \rightarrow$ dimensionless

$$\frac{t}{a} \rightarrow \text{dimensionless}$$

$$a \rightarrow [T^1]$$

hence, a^x should be dimensionless.

$$\Rightarrow \quad \boxed{x = 0}$$

Ques.: If energy E, velocity v & time are taken as fundamental units then find dimensional formula for surface tension

Solns.:

$$S \propto E^a{}_0,\ V^b{}_0,\ t^c$$

$$[MT^{-2}] = [ML^2T^{-2}]^a[LT^{-1}]^b[T]^c$$

Compare the power of M, L, T

$$\Rightarrow \quad a = 1,$$

$$\Rightarrow \quad 2a + b = 0$$

$$b = -2$$

$$\Rightarrow \quad -2a - b + c = -2$$

$$S \propto E\ v^{-2}t^{-2}$$

$$S \propto \frac{E}{v^2 \cdot t^2}$$

$$S = K\frac{E}{v^2 \cdot t^2}, \text{ where } K = \text{dimensionless constant}$$

Ques.: The value of gravitation constant is

$G = 6.67 \times 10^{-11}\ NM^2\ kg^{-2}$. Convert into CGS system of units.

Soln.: $6.67 \times 10^{-11}\ (10^5\ \text{dyne})(10^2\ \text{cm})^2\ (10^3\ \text{g})^{-2}$

$6.67 \times 10^{-8}\ \text{dyne cm}^2\ \text{g}^{-2}$

Ques.: Using the method of Dimensional analysis, check the dimensional correctness of the relations $v = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ where l is length, v is frequency measured in $\sec^{-1}$, T is tension in Newton, μ is mass per unit length.

Soln.:

$$[\text{LHS}] = [T^{-1}]$$

$$[\text{RHS}] = \left[\frac{1}{L}\sqrt{\frac{MLT^{-2}}{ML^{-1}}}\right] = \left[\frac{1}{L}\sqrt{L^2T^{-2}}\right] = [T^{-1}]$$

Since, [LHS] = [RHS], the equation is correct dimensionally.

Ques.: Write the dimensions of a/b in the relation $F = a\sqrt{x} + bt^2$ where, F is force, x is distance and t is time.

Soln.: Since,

$$F = a\sqrt{x} + bt^2,\ \left[a\sqrt{x}\right] = [bt^2]$$

$$= [a]\left[L^{\frac{1}{2}}\right] = [b][T^2]$$

$$= \frac{[a]}{[b]} = \frac{[T^2]}{\left[L^{\frac{1}{2}}\right]}$$

$$\therefore \quad \left[\frac{a}{b}\right] = \left[L^{-\frac{1}{2}}T^2\right]$$

Ques.: The velocity v of a particle depends upon time t according to the relation

$$v = a + bt + \frac{c}{d+t}$$

Write the dimensions of a, b, c and d.

Soln.:

$$[v] = [a] = [bt] = \frac{[c]}{[d+t]} \quad \ldots(1)$$

Also,

$$[d + t] = [d] = [t] \quad \ldots(2)$$

From (2),

$$[d + t] = [d] = [T]$$

Substituting in (1), we get $[LT^{-1}] = [a] = [b][T] = \frac{[C]}{[T]}$

$$\therefore \quad [a] = [LT^{-1}]$$
$$[b] = [LT^{-2}]$$
$$[c] = [L]$$
$$[d] = [T]$$

Ques.: If force (F) and density (d) are related as $F = \dfrac{\sqrt{a}}{(2b+\sqrt{d})} + 3c$, calculate the dimensions of a, b and c.

Soln.:
$$[F] = \frac{[\sqrt{a}]}{[2b+\sqrt{d}]} = [3c] \quad ...(1)$$

Also,
$$[2b+\sqrt{d}] = [2b] = [\sqrt{d}] \quad ...(2)$$
$$[2b+\sqrt{d}] = [b] = [\sqrt{ML^{-3}}]$$
$$\therefore \quad [2b+\sqrt{d}] = [b] = [M^{1/2}L^{-3/2}]$$

Substituting in (1), we get
$$\left[MLT^{-2} = \frac{[\sqrt{a}]}{[M^{1/2}L^{-3/2}]} = [c]\right]$$
$$= [\sqrt{a}] = [M^{3/2}L^{-1/2}T^{-2}]$$
$$\therefore \quad [a] = [M^3L^{-1}T^{-4}]$$
$$\therefore \quad [b] = [M^{1/2}L^{-3/2}],$$
$$[c] = [MLT^{-2}]$$

Ques.: Given that the time period (T) of oscillation of a gas bubble from an explosion under water depends upon pressure (P), density (d) of water and total energy (E) of explosion, find dimensionally a relation for T.

Soln.: Let $T = k \cdot p^a d^b E^c$ where,
K is a dimensionless content.
Writing the equation in dimensional form, we have
$$[T] = [ML^{-1}T^{-2}]^a[ML^{-3}]^b[ML^2T^{-2}]^c$$
$$= M^0L^0T^1 = M^{a+b+c}\ L^{-a-3b+2c}\ T^{-2a-2c}$$

On equating powers on both side, we get
$$a + b + c = 0,\ -a - 3b + 2c = 0,\ -2a - 2c = 1$$

On solving, we get $a = -\frac{5}{6},\ b = \frac{1}{2},\ c = \frac{1}{3}$

$\therefore$ $T = k\ P^{-5/6}\ d^{1/2}\ E^{1/3} = k\frac{d^{1/2}E^{1/3}}{P^{5/6}}$

Ques.: If velocity, force and time were chosen as fundamental quantities, and their dimensions are V, F and T respectively, what are the dimensions of mass?

Soln.: Let $[M] = [V^a F^b T^c]$

Then,

$$[M] = LT^{-1}]^a\ [MLT^{-2}]^b\ [T]^c$$

$$= [M^1L^0T^0] = [M^b\ L^{a+b}\ T^{-a-2b+c}]$$

On equating powers on both sides, we get

$$b = 1,\ a + b = 0,\ -a - 2b + c = 0$$

On solving, we get

$$a = -1,\ b = 1,\ c = 1$$

$\therefore$ $[M] = [V^{-1}F^1T^1] = [V^{-1}FT]$